THE EFFECT OF A PROBLEM BASED LEARNING APPROACH ON THE TEACHING AND LEARNING OF COMPOSITION AND INVERSES OF FUNCTIONS IN A FOUNDATION PROGRAMME

by

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DECLARATION

By submitting this dissertation, I declare that the entirety of the work contained therein is my own, original work, that I am the sole author thereof (save to the extent explicitly otherwise stated), that reproduction and publication thereof by Stellenbosch University will not infringe any third party rights and that I have not previously in its entirety or in part submitted it for obtaining any qualification.

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ABSTRACT

The purpose of the study was to investigate *The effect of the Problem-Based Learning Problem Based Learning (PBL) approach in the teaching of composition and inverse functions in a foundation programme*. PBL is a philosophical approach to teaching and learning where problems drive the learning. The study was important because it was trying to find out if PBL can improve students' performance in compositions and inverses of functions at the bridging course for undergraduate mathematics at Oshakati Campus.

The study intended to come up with a PBL model suitable for FP mathematics in the teaching of compositions and inverses of functions. The study was done on Science Foundation students who are registered for FP. Eighty students were randomly selected from the foundation students registered for the 2013 academic year. The students were randomly assigned into the experimental and the comparison groups of 40 each. In this study the comparison group of the Foundation students was predominantly taught through the traditional lecture approach while the experimental group was predominantly taught using a hybrid PBL approach.

The study also attempted to establish the students' perceptions with regard to the relevance of inverses and compositions of functions as a concept in a topic that determines their academic destination. It also attempted to ascertain how the PBL approach could best be implemented in order to improve FP students' understanding of inverses and composition of functions; how Bridging course for undergraduate mathematics (FP) students experience the PBL approach in the teaching and learning of inverses and composition of functions compared to those who are taught using the lecture method and how FP students' performance on inverses and composition of functions as a result of their PBL experience compare to those who are taught using the lecture method.

This study used the concurrent nested mixed methods (qualitative and quantitative) research designs. A quasi experimental design was adopted through the administration of a pre-post-test on experimental and comparison groups. The other designs or methods included a questionnaire survey, focus group interviews, non-participant lesson observation and a group research project on compositions and inverses of functions. The experimental group was then mainly taught through a hybrid PBL approach while the comparison group mainly through the lecture approach for a period of three months.

The findings of this research study showed that experimental group students performed significantly better in the overall results analysis but there were no significant differences in performance between the two groups for some Hypothetical Learning Trajectory (HLT) domains on compositions and inverses of functions.

It is recommended that PBL should be implemented in the other foundation programme subjects. However, the role of the conventional teaching approaches cannot be undermined in the teaching and learning of compositions and inverses of functions since the students who were taught using this method also improved their performances, and as such these conventional teaching approaches should be used together with PBL in order to get the best results on FP students' mathematics performance. This study recommends further research on how PBL can be implemented in other FP subjects. This study also recommended that PBL should be implemented right at the beginning of the year when the FP students start their classes in the foundation programme.

OPSOMMING/SAMEVATTING

Die doel van die studie was om die effek van die probleemgebaseerde leer (PBL) benadering in die onderrig van die samestelling en inverse funksies in 'n Stigting program te ondersoek. PBL is 'n filosofiese benadering tot onderrig en leer waar probleme ry die leer. Die studie is belangrik omdat dit probeer het om uit te vind of PBL kan studente se prestasie in komposisies en inverses van funksies te verbeter by die Stigting Program op Oshakati-kampus.

Die studie bedoel om vorendag te kom met 'n PBL model wat geskik is vir fondament in die onderrig van komposisies en inverses van funksies. Die studie is gedoen op Science Foundation studente by Oshakati-kampus van die Universiteit van Namibië. Tagtig studente is lukraak gekies uit die fondament studente wat geregistreer is vir die 2013 akademiese jaar. Die studente is ewekansig toegewys in die eksperimentele en die vergelyking groepe van 40 elk. In hierdie studie is die vergelyking groep van die Stigting studente is hoofsaaklik geleer word deur die tradisionele lesing benadering terwyl die eksperimentele groep was hoofsaaklik geleer met behulp van 'n hibriede PBL benadering.

Die studie het ook probeer om vas te stel uit wat die studente se persepsies met betrekking tot die toepaslikheid van inverses en komposisies van funksies is soos 'n konsep in 'n onderwerp wat bepaal hul akademiese bestemming. Dit het ook probeer om vas te stel hoe die PBL benadering kan die beste om FP studente se begrip van inverses en samestelling van funksies te verbeter geïmplementeer word; hoe FP studente die PBL benadering in die onderrig en leer van inverses en samestelling van funksies in vergelyking met diegene wat geleer is met behulp van die lesing metode en hoe FP studente se prestasie op inverses en samestelling van funksies as 'n gevolg van hul PBL ervaring vergelyk met dié wat geleer is met behulp van die lesing-metode.

Hierdie studie gebruik om die konkurrente geneste gemengde metodes (kwalitatiewe en kwantitatiewe) navorsing ontwerpe. 'N quasi eksperimentele ontwerp is aangeneem deur die administrasie van 'n pre-na-toets op eksperimentele en vergelyking groepe. Die ander ontwerpe of metodes het 'n vraelys opname, fokusgroeponderhoude, nie-deelnemer leswaarneming, en 'n groep navorsingsprojek oor komposisies en inverses van funksies. Die eksperimentele groep is dan hoofsaaklik geleer deur middel van 'n kruising PBL benadering terwyl die vergelyking groep hoofsaaklik deur die lesing benadering vir 'n tydperk van drie maande.

Die bevindinge van hierdie navorsing het getoon dat die eksperimentele groep studente uitgevoer aansienlik beter in die algehele resultate analise, maar daar was geen betekenisvolle verskille in prestasie tussen die twee groepe vir 'n paar MTT gebiede op komposisies en inverses van funksies. Die studie het ook bevind dat PBL aan die begin van die jaar reg geïmplementeer moet word wanneer die FP studente begin hul klasse in die fondament program.

Dit word aanbeveel dat PBL in al die ander fondament program vakke moet geïmplementeer word. Tog kan die rol van die konvensionele onderrig benaderings nie ondermyn word in die onderrig en leer van komposisies en inverses van funksies, en as sodanig die konvensionele onderrig benaderings moet saam met PBL word gebruik om die beste resultate op FP studente se wiskunde prestasie te kry . Hierdie studie beveel aan verdere navorsing oor hoe PBL in 'n ander fondament program vakke geïmplementeer kan word.

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ACRONOMYS

DNEA	Directorate of National Examinations and Assessment
FP	Foundation Programme
MAF1100	Mathematics Science Foundation course code 1100
MEC	Ministry of Education and Culture
MoE	Ministry of Education
PBL	Problem Based Learning
NPC	National Planning Commission
SPSS	Statistical Package for Social Sciences
UNAM	University of Namibia
RME	Realistic Mathematics Education
HLT	Hypothetical learning Trajectory
SWAPO	South West Africa Peoples' Organisation
ASPs	Academic Support Programmes
NAMCOL	Namibian College for Open Learning

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CHAPTER ONE

INTRODUCTION AND ORIENTATION

1.1. BACKGROUND OF THE STUDY

The Science Foundation (FP) programme at the University of Namibia was established in 2005 by the Senate after the realization that there were few graduates in science related professions in Namibia. One of Namibia's national goals is the attainment of Vision 2030, which anticipates that the country will be developed and industrialized by the year 2030 (National Planning Commission [NPC], 2003). The realization of this noble national goal is impossible if Namibia cannot produce its own manpower in mathematics and science related fields. The aim of the foundation programme is to lay a sound basis for future study, thereby bridging the educational gaps between teaching at the University and high school (Naukushu, 2012).

Hence the Namibian government entrusted the University of Namibia to come up with a programme that specializes on equipping students with the relevant scientific skills. The University of Namibia developed all the necessary materials through curriculum specialists in Mathematics and Science and came up with the foundation programme which has been offered at the Oshakati Campus since 2005 (Ngololo & Kapenda, 2012). Ngololo and Kapenda (2012) further state that the foundation programme aims at increasing access to the science related faculties at the University of Namibia. The programme further aims at broadening access to higher education to previously disadvantaged and marginalized groups and afford them opportunities to enrol in the Science related degree programmes at the university (Portgieter, Davidowitz, & Mathabatha, 2007). Many of the few students who gain direct access to the university do so without the critical knowledge and skills required to competently understand the subject matter in Sciences and Mathematics in the first year. This is attributed to the fact that learners graduating from historically disadvantaged schools often do not attain the same level of understanding and educational achievement as the ones that attend well-resourced schools.

In the Namibian setting, there is a gap between high school mathematics and university mathematics. The Namibian Mathematics curriculum does not prepare students fully for university mathematics (Miranda, Nakashole, & Chirimbana, 2013). In the bridging course for undergraduate mathematics, classroom instruction should objectively be focused not just towards knowledge dissemination, but towards reflective, analytical, creative and practical thinking with a solid knowledge base. Students at all levels learn better when they think in order to learn (Van der Flier, Thijs, & Zaaman, 2003). Such learning takes place when they take into account their diverse learning and thinking styles (Sternberg, 2008). The purpose of learning should be that of learning how to learn and learning how to think in order to keep pace with the incessantly changing knowledge demands of the 21st century. To be more specific, learners should be equipped with the skills that will allow them to work in dynamic work environments. The bridging course for undergraduate mathematics at Oshakati Campus of the University of Namibia is designed in such a way that it prepares students as fully as possible for university education. Thus the type of education offered at the bridging course for undergraduate mathematics is intended to equip these learners fully with the proper academic and professional skills for them to grow from novices to experts in the work environment. The lecture-based learning approaches that have dominated most of the tertiary education systems and have been in use for so many years are known to be content-driven (Amutenya, 2002). In the past these teaching approaches have been seen to be appropriate and were preparing the students at tertiary level to meet the professional demands of the old days where the emphasis was not to make students autonomous thinkers who should apply their education to solve day to day real life problems.

However, these methods have fallen short of the needs and demands of the modern workplace and, as such, some more relevant teaching methods that closely correlate with the workplaces of today are being advocated for (Albanese and Mitchell, 2004). Conventional instructional methods in the teaching of mathematics have been found not to fully prepare students with the necessary professional attributes and technical know-how they need in handling the demands of the future work environments. The current traditional teaching approach makes students inert recipients of mathematics knowledge and encourages rote learning since the methods are teacher-centred. Miflin (2004) has stated that in the lecture method, the lecturer tends to teach mechanistically and standard type of problem and solution with minimum involvement of students. By this he means that the lecturer is the one responsible for the delivery of the content to be leant in class and teaches specific problems in a specific way. The lecture method is a teacher-centred approach in which the teacher plays the leading role in the information transfer process. In this method the teacher is regarded as the expert, authority figure and main source of knowledge and the focal point of all the information is the class (Colliver, 2001). The compositions and inverses of functions in the FP of the University of Namibia Oshakati Campus are abstract concepts which are normally lectured to the students. Students' involvement in the learning process is very minimal such that at the end of the day they can manipulate inverses and compositions of functions without necessarily understanding their real life applications. The dominant teacher-centred teaching method, especially on compositions and inverses of functions in the bridging course for undergraduate mathematics at Oshakati Campus does not provide active learning environments for students.

However, irrespective of the fact that traditional teaching approaches fail to motivate students in becoming active learners, they are still being used widely in many tertiary institutions due to their main economic benefit of allowing large numbers of students to be taught and produce huge numbers of tertiary graduates over a short period of time. Though it can be acknowledged that these traditional teaching methods have passed the tests of time since they have, and are still being used at the FP, their employment in the teaching of compositions and inverses of functions encourages rote learning. This is because students just memorise procedures and apply what they have memorised without necessarily understanding their practical relevance and applications. As the rate of change in knowledge continues to increase this notion does not make students autonomous thinkers who can apply their academic knowledge to solve problems in their careers and, for this reason, educational systems must develop skills for lifelong learning (Schwalbach, 2006).

1.1.1 Namibian education system before independence

Amukugo (1993) indicates that historically, the African education system was both formal and informal. In the informal educational system learning took place through observing: the younger generation observed their siblings and elderly members of the community whereas the formal education meant institutionalized education, and carefully planned educational programmes. This was the educational set up of Africa before the coming of the missionaries. Mbamba (1982) asserts that the history of formal education in Namibia, like everywhere else on the African continent, goes back to the early activities of the first missionaries. Moreover, Amukugo (1993) adds that such an ideology might suggest that western education in Namibia and in Africa in general, was introduced into an educational vacuum. The type of mathematics which was taught those days was basic arithmetic but greater emphasis was on bible knowledge and craft education and needlework (Mbamba, 1982). Amukugo (1993) further states that the common features in the Namibian informal education before independence were that an individual would come to know his or her duties, rights and responsibilities as well as those of others.

The German colonial education was built on the foundation laid by missionaries the pioneers of which were the London and Wesleyan societies in 1805, Rhedish Missionary society in 1842 and the Finnish Missionary society in 1870 (Amukugo, 1993).The education which was provided by the missionaries to the Namibian people was shaped by both missionaries' religious motivation to convert Namibians into Christianity and the political motivation to convince Africans of the need for state protection. The resistance to conversion by Namibians as stated by Amukugo (1993) prevented the missionaries from establishing successful educational programmes. The education and training which the Namibian people received around 1907 was only sufficient for them to perform manual tasks (Mbamba, 1982; Amukugo, 1993). This type of system was intentionally designed to make sure that the Black Namibian people would remain in a state of educational inferiority and submit to the needs of their white colonisers. As clearly stated by Amukugo (1993), the German colonial administrators aimed:

> "…to educate them… to inculcate such mischievous and intolerable ideas as democracy, brotherhood of man, human freedom and the like." (p. 37).

Good education was provided for the whites, and their educational services continued to expand especially after the 1904-1907 World war. More new schools and hostels for the whites were built between 1908 and 1909. Generous grants were given to white parents for them to be able to send their children to the best boarding schools. School attendance was compulsory for all white students who were staying within a radius of four kilometres. The African education system, however, never progressed beyond simple literacy and bible study during the colonial era (Mbamba, 1982).

The German colonial rule of Namibia ended in 1918 with the defeat of Germany in World War 1. Namibia was placed under South African rule in 1919. South Africa's colonial power over Namibia was strengthened (Amukugo, 1993). In 1919 South Africa placed the education of Namibia under state comparison. This meant that little funding was going to be given to Namibia to expand its education system for blacks. In 1921 a state educational proclamation which allowed provision of schools for the whites and schools for the blacks was made. During that time the education for the Namibian people would last from several months to four years where the subjects being emphasized were writing in the mother tongue, elementary arithmetic, singing, and hygiene, manual instruction in woodwork, metalwork, gardening, and building for boys; and for the girls it was needlework, basket making and housework (Clegg, 1989). This form of syllabus was just a carbon copy of the German syllabuses which just prepared the black Namibians for simple manual professions and not equip them with technical jobs which were reserved for the whites (Mbamba, 1982). Inequalities in educational opportunities and expenditures between the blacks and the whites continued to dominate the Namibian educational system. The parents for the Namibian children had to pay fees for their children's education while South African whites were not made to pay.

The period 1948-1975 was called the Apartheid era. During this period, the Bantu education system was formed. Amukugo (1993) states that the curriculum for the Bantu education system included reading and writing in the mother tongue, singing, handicraft, elementary arithmetic, environmental studies, hygiene, physical training, religious instruction, English and Afrikaans. In an effort to maintain social, economic and academic segregation, the architects of apartheid fragmented Namibia into eleven education authorities based upon ethnicity (Dunn, 2003). This system was designed to inculcate racism, the philosophy underlying the apartheid-era education system and was premised on the notion that Blacks were incapable of learning mathematics and science (Clegg, 1989).The education system was reformed in such a way that training was given to people according to the opportunities they had in life and people who believed in equality were regarded as bad teachers for the natives. The medium of instruction which was used for the Black students during Apartheid was Afrikaans. It was argued that their masters would either be English or Afrikaans speaking. They would not make good servants if they did not understand the master's language. The length of schooling for Blacks and Whites was made the same but from 1976, the schooling for black Namibian children was compulsory up to the third year only. The Bantu educational system (education for the Black Namibian) offered many subjects at primary school but few of them were conducive to an all-round, emotional and intellectual development of the pupils with little emphasis on mathematics education (Clegg, 1989). At the end of the seven years of primary education in the Bantu system, all that the pupils could do was to care for white peoples' homes, gardens and farms with little knowledge about arithmetic, reading and writing (Amukugo, 1993). Thus it can be argued that the Bantu education content was only preparing the Africans to join a semi-skilled labour force and fill the lower positions in the labour market. This educational system ensured that the colonial power would be provided with a steady supply of semi-literate, subservient farm and house labourers (Christie & Collins, 1984).

This system was also examination driven; it further marginalized and disadvantaged indigenous Namibian students by ensuring that only a small number of privileged students would succeed. Basing on all this evidence, Amukugo (1993) concludes that South Africa as a colonial power used education as an instrument for white domination over Africans and introduced Bantu education as no more than a well-planned state instrument for the production of cheap labour.

In 1980 the Namibian education system was reformed, presumably to meet the needs of all the Namibian people. This reform was done by the Advisory Committee for Human Sciences Research (ACHSR). The process included the introduction of a pre-primary programme, which was previously not available to the Namibian natives. The primary phase was to consist of a six-year course instead of seven. Tertiary institutions were mandated to provide diploma courses and eventually degrees with little emphasis on mathematics (Dunn, 2003).The focus was not to make the Black Namibians mathematically literate, but to train them for simple non-technical jobs while they are under the supervision of a their White masters.

1.1.2. Namibian Education system post-independence

Following a protracted armed struggle for liberation, Namibia gained its independence from South Africa on 21 March 1990. The apartheid regime left as its legacy an educational system shaped by divisive and dehumanizing imperialist policies. The South West Africa Peoples' Organization (SWAPO) recognized that knowledge and power had been connected and interrelated in the achievement of subjugation and thought of reforming Namibia's education system after independence. After independence, changes in the educational system were pertinent if Namibia was to resolve the cumulative social, academic, ethnic and professional imbalances brought about by the Germany and South African colonial systems. SWAPO embarked on an educational reform programme that served the interests of the Namibian people. This system had a slight improvement in the emphasis on the teaching and learning of mathematics (Nambahu, Shikongo, & David, 2005). Amukugo (1993) states that one of the policies of SWAPO was the provision of work-oriented, comprehensive education and training for illiterate and semi-illiterate adults (with a literacy component built in) at SWAPO schools. Another core aim was to offer a free and universal education system for all Namibians from primary through secondary to university level which had to be achieved by training the required number of teachers and educationists. The movement to reform mathematics education in Namibia began in the mid-1980s in response to the documented failure of traditional methods of teaching mathematics, to the curriculum changes necessitated by the widespread availability of computing devices, and to a major paradigm shift in the scientific study of mathematics learning (Amukugo, 1993)

As SWAPO dismantled the discriminatory and inequitable Bantu educational system, the newly independent Namibian government embarked on and adopted a new praxis. The Namibian government mandated reforms based on the goals of access, equity, quality and democracy (Ministry of Education and Culture (MEC), 1993). The new curriculum offered a wide range of subject options that included: English, Civics, Mathematics, Hygiene, Science, Economics, Bookkeeping and Commerce.

Amukugo (1993) asserts that a fair comparison on the two types of education systems offered to the Namibian people before and after independence clearly shows that the education system for SWAPO provided a relatively high level of quality as compared to that of the South Africans and the Germans. There was an improved enrolment in Science and Mathematically related professions which were previously reserved for the Whites. However, in a newly independent Namibia, education was seen and understood as a social and cultural reconstruction (Bowe, Ball, & Gold, 1992). Moreover; educational change has consequences for social justice, equity and democracy. Therefore there was an improved access to basic and higher education during the SWAPO education system

Achieving access and quality education has been a challenge to the Namibian government since independence (Naukushu et al., 2012). Naukushu et al (2012) further asset that despite the government's efforts to alleviate the challenge of lack of access and quality in the provision of education, many schools, mainly in rural areas, still do not have some resources and they are not as well equipped as schools in the urban areas. Learners graduating from the schools with poor resources persistently do not attain the same level of understanding and educational achievements required for admission into and success in tertiary institutions compared to those that attend well-resourced schools (especially in Mathematics and Sciences).

The National Planning Commission (NPC) (2003) indicates that 60% of the Namibian population lives in the rural areas of the Northern and Central regions (i.e. Oshana, Oshikoto, Omusati and Ohangwena regions). Most of the learners from these regions live in what the Ministry of Education terms "poor educational background". The inequities in education due to the colonial legacy also make it difficult for the schools to be at the same standard throughout the country. Therefore, the University of Namibia at its Oshakati Campus re-conceptualised and transformed its Access Programme into a Science Bridging course for undergraduate mathematics now known as the Foundation Programme.

1.1.3. Brief history of SFP at UNAM

The origin of FPs was traced by Van der Flier, Thijs, & Zaaman (2003) more than 30 years ago when a number of tertiary institutions in the UK, America and other western countries realized the need of implementing foundation programme to widen the access to an ever-growing, education-hungry population. Uugwanga (2006) notes that this trend spilled over to Africa and a number of African universities now offer access to potential students by way of FPs or bridging programmes. Mabila et al. (2006) observes that in South Africa for instance, the pathways to Higher Education program aimed to increase graduation rates at public universities by giving promising students in the Mathematics, Science, and Agricultural fields the opportunity to learn, excel and contribute positively to economic growth. Participating institutions of the Pathways Program in South Africa are North West University, University of Fort Hare, University of Limpopo, University of Venda, University of Stellenbosch with a cumulative number of pathway students numbering nearly 2300. The University of Namibia also participated in the Pathways Program in its earlier years (Uugwanga, 2006).

Though the FP was introduced in Namibia in 2005, its history can be traced back to the early 1980s (Kloot, Case, & Marshall, (2008)). It is interesting to note that it took more than 14 years after independence before the introduction of the FP in Namibia. It was in the early 1980s, in the latter days of the apartheid era, within a fragmented education sector that FP emerged in Namibia. The first ‘bridging’ programmes at the white, English-medium universities were a means of academic support that was offered to assist a small number of black students. These initiatives can be considered an early form of FP. In many cases they were financed by corporate capital that was voicing concerns about a black skills shortage in the job market. Kloot et al. (2008) further argue that they were primarily interested in the reproduction of capitalist social relations rather than social transformation.

As their name suggests, FP are bridging programmes focussed on ‘filling the gaps’ left by inadequate schooling in the hope that students would then be able to cope with the demands of tertiary study. Kapenda and Ngololo (2012) point out that Academic Support Programmes (ASPs) were a ‘reactive response’ to the problem of poor academic performance of black students and that they started with ‘no or little theoretical underpinnings’ (p. 11). The aim was simply to assist students from inferior schools in predominantly white tertiary institutions. Taking an extra year to bridge the ‘educational gap’ was considered a necessary measure.

The attitude of the university ‘mainstream’ – the traditional programme offerings – was very much ‘business as usual’ while it was left to academic support to get on with the job of preparing disadvantaged students for an institution that was itself to remain unchanged (Naukushu et al., 2012). Growing criticism towards these Academic Support Programmes (ASPs) prompted universities to look more seriously at the issue of under-prepared students. At the FP student-centred learning, the use of English rather than Afrikaans as the medium of instruction, and a holistic consideration of students’ prior knowledge and experiences and their intellectual, emotional, social, physical, aesthetic, moral and spiritual development are the cornerstones of a transformed Namibian education system (Dunn, 2003). The redesigned secondary school mathematics school curriculum has been structured based upon a constructivist view of knowledge, learning competencies in content areas, developing a reflective attitude, and promoting creative, analytical and critical thinking (National Institute for Educational Development, 1998). Irrespective of all these improvements, measures, from the researcher’s experience, students’ understanding of compositions and inverses of functions remains a great challenge.

Achieving access and quality in higher education has been a challenge to the Namibian government since independence (Naukushu et al, 2012). Despite the government’s efforts to alleviate the challenge of lack of access and quality in the provision of education, many schools, mainly in rural areas, still do not have the same resources and are not as well equipped as schools in the urban areas (Dunn, 2003; Naukushu et al., 2012; Rollnick and Holtman, 2010).

Moreover, the University of Namibia faces significant challenges including responding to the needs and demands of the disadvantaged society, often living in the rural areas and far outlying regions. SFP were introduced in an attempt to redress these inequalities of the past; resolving the constraints to the expansion of the University of Namibia caused by weak academic preparation of students and widening access to tertiary education and at the same time maintaining the standards and quality of its programmes (Ngololo and Kapenda, 2012).

Mabila, Malatje, Addo-Beniako & Kazeni (2006) argue that bridging course for undergraduate mathematics are viewed as vehicles towards achieving this goal and consequently are the priority areas of the education sector in the country. There is thus a greater need to increase the quantity and quality of graduates in the areas of Science and Mathematics (Handelsman, Miller, & Pfund, 2007).

Foundation Programmes are defined by Kloot et al. (2008) as academic developmental programmes meant to assist disadvantaged students and underprepared students to enrol in tertiary instituitions. The term foundation emerged to describe a set of courses that attempt to lay the necessary academic foundation for tertiary education (Holtman, Marshall, & Linder, 2004). De Beer (2006) points out that the FP lays a sound basis for future study thereby bridging the educational gaps between teaching at the university and high school.

Amutenya (2002) contents that there existed three separate education systems in Namibia i.e. the education for Blacks, Whites and Coloureds. The Black majorities were believed to be incompetent in Mathematics and Science related subjects beyond the elementary level. Such injustices in the education system continued to create a critical shortage of manpower in science related professions like engineering, Mathematics and Science teaching, nursing, medicine and pharmacy, thereby imposing pressure on the Namibian Government to import specialist personnel in these fields from other African countries like Zimbabwe, Zambia, Kenya, Tanzania etc. Therefore the formation of the FP was the Namibian Government's response to the previous educational injustices the Namibians have suffered (Ngololo & Kapenda, 2012; Naukushu et al., 2012). It is a relevant concept for an industrialising and developing nation such as Namibia. This view is further defined in the Education and Training Sector Improvement Programme (ETSIP) document that pronounces the Namibian strategy for improving education. It is envisaged that the new strategy for improving education will foster broader benefits that are critical to development, including poverty reduction and social equity through education programmes that will allow Namibia supply its on manpower. Hence the FP serves as a tool that will provide the necessary academic skills (Naukushu et al, 2012; Mabila et al, 2006; Rollnick and Holtman, 2010).

There is ample evidence from the Directorate of National Examinations and Assessment (DNEA) (2009) that there is a high failure rate due to the failure of secondary education system at all levels to meet the demands of higher education in both aspects of quality and quantity. This was observed particularly in all areas of Mathematics and Science, hence it was deemed necessary to establish a platform that prepares students in Mathematics, Sciences and English to enable them to cope with the demands of higher education in Science and Mathematics disciplines (Holtman, Marshall, & Linder, 2004).

The foregoing literature by (Naukushu et al, 2012; Uugwanga, 2006; Mabila et al., 2006; Holtman et al., 2004; Rollnick and Holtman, 2010) affirms that all FPs provide an alternative entry route into universities by expanding admission policies and most importantly to help those students that do not meet the standard set admission criteria. The main focus of FPs in Africa and specifically in Southern Africa (UNAM included) is to therefore help formerly marginalized and educationally disadvantaged students to gain entry into universities to follow degree and diploma courses of their choice. As a result, these students would in due course make an impact on the socio-economic status quo of individuals, the community and the country (Portgieter, Davidowitz, & Mathabatha, 2007).

1.1.4. Current state of teaching Mathematics at the Oshakati Campus FP

At the Science Foundation Programme (SFP) of the University of Namibia, Mathematics is taught in three different classes identified as classes 1-3. The students are assessed using Continuous Assessment (CA) which constitutes 60% of the total assessment and then an examination which constitutes 40% of the overall pass mark. A student is said to have passed if he or she achieves an overall pass mark of 60% in all the five subjects which are Mathematics, Chemistry, English, Biology and Physics. In Mathematics, students are assessed using 5 formal tests, 5 home works and 5 assignments which are spread throughout the academic year, which is in two semesters (UNAM, 2005). The first semester runs from February to June and the second semester from July to November. All SFP subjects are year courses i.e. examinations are only written at the end of the year in November. The main teaching approach which is used at the SFP is the traditional lecture method, which is done in class sizes of less than 50 (Chirimbana, 2013). However, studies by (Chaplin, 2007; Handelsman et al, 2007) show that lectures are not as effective in ensuring retention and hence the lecture is the most inefficient way of teaching to undergraduate students.

Although students in the FP may sometimes be taught by more innovative, student centred methods, once they enter the mainstream of undergraduate students, they need to take responsibility for their own learning in contexts where the lecture is the main mode of delivery (Rollnick and Holtman, 2010). At the SFP the traditional lecture has been pepped up by technology, and yet this is often a mixed blessing. Kamati (2012) found that the students she taught effective presentation skills felt insulted and neglected by lecturers who read their lecture from PowerPoint slides. Kamati further points out that when lecturers replaced the chalkboard by the overhead projector, they did not change their teaching method. Similary in a SFP PowerPoint does not change a bad presentation into a good presentation or an ineffective presenter into a good presenter. On this note warns that PowerPoint should enhance a presentation, not replace effective communication.

The SFP at UNAM is strategically designed to increase the number of Science and Mathematics degree intakes. This programme draws students from a larger pool of high school leavers with poor grades in their grade 12 examinations that did not make it to UNAM because of the school environment they are in. Some of the challenges faced by Namibian students include among others: floods, HIV/AIDS effects, lack of qualified teachers and lack of educational resources (Chirimbana, 2013). These students are often from lower socio-economic groups, often isolated rural communities, and have inadequate exposure to career guidance and counselling. They are financially disadvantaged and are not usually exposed to science outside of the classroom because they do not read magazines or watch televisions (Potgieter et al, 2005)

Students who wish to pursue science related professions are required to have at least a C symbol or better in either core mathematics or a D symbol in Extended Mathematics for them to be enrolled in tertiary institutions in Namibia. Therefore the SFP is set to meet this goal and resolve the manpower shortage in various science related fields of the Namibian economy.

The mathematics syllabus covers the depth and breadth of mathematical concepts ranging from algebra, trigonometry, geometry, measurement, mensuration, consumer arithmetic, functions, sets, definition of numbers etc. Functions, which are the focus of this study, are taught during the second semester after the students' understanding of algebraic concepts is now at an advanced stage (UNAM, 2005). The covered function concepts include: definition of a one-to-one function, evaluation of a function, domain and range of a function, graphs of different functions and finally inverse and composition of functions which are the mathematical objects of investigation in this study.

1.1.5. Motivation for the choice of composition and inverses functions.

Over the years the researcher has come to realise that human life can be defined as a function of many variables. Being a teacher, the operations of my life are a function of how I relate to my workmates at the work place, how I relate to my church mates and how I budget for my family's needs, food, clothing, school fees, medical expenses, etc. From a philosophical perspective, the researcher has concluded that composition and inverse functions form such an important component of human life in that they relate a man to another man, and a man to nature and vice versa. Human life function as a result of many variables embedded in one; relationships with other people, relatives, friends, food, work, church, etc., and also how these variables also relate to humans. Using my experiences as a teacher, I realized that students' success in mathematics is a function of how hard work, how talented or fortunate they are, and how attentive they are in class. Based on these, I found compositions and inverses of functions to be of paramount importance to be the focus of my study.

As a lecturer of mathematics at the SFP of the University of Namibia, I started encountering more and more functions in the curriculum: quadratic, logarithmic, trigonometric, polynomial, exponential, compositions and inverses etc. With these real facts, I finally concluded that human life cannot be detached from compositions and inverses of functions; they are real and practical and relevant in the lives of every individual. As a result of this, my topic interest shifted towards studying functions and how their understanding can be enhanced through the PBL approach, which is an approach that also uses relevant real world problems. The researcher concentrated only on two aspects of the function; which are the composition and inverse functions with the purpose of improving students' performances in this topic and laying a solid foundation for first year calculus courses in particular.

1.2. SIGNIFICANCE OF THE STUDY

Students' performances in compositions and inverses of functions are very low at SFP. This study is therefore significant in shedding light on how the PBL approach can be effectively implemented to enhance students' conceptual understanding of FP mathematics in general and functions in particular, in preparation for access with success in higher education STEM careers.

The implementation of the PBL approach in a Namibian Bridging course for undergraduate mathematics is a relatively new concept at Oshakati FP; therefore the findings from this study will help the university management and academics in paving ways for PBL implementation in the Namibian educational setting. Apart from that, the findings from this study may also be used by the Ministry of Education (MoE) policy developers to design PBL friendly policies which may be used to guide teachers in the implementation of PBL in Namibian schools. Successful completion of this study may give insight to other subject specialists who may want to implement PBL in the teaching and learning of their various subjects.

1.3. PROBLEM STATEMENT

UNAM (2012) states that only students with a minimum of an E symbol in Mathematics, E in English, D in Physical Science and a D symbol or better in any other subject are eligible to enrol for the SFP. It is after pursuing the above mentioned five subjects that they are able to enrol in Mathematics and Science related fields within the university after successfully completing the programme.

In the SFP each student is assigned a lecturer who advises him/her on their progress. The lecturer also finds out from the student any social challenges which the student may face (UNAM, 2005). Despite all these efforts by the university, students in the SFP do not do well in Mathematics as compared to the other subjects such as Physics, Chemistry, English and Biology (Naukushu and Chirimbana, 2012). The students usually get marks below 60%, thus disqualifying them from proceeding to their desired programmes and ending up enrolling for diplomas or certificates (Chirimbana, 2013). With regard to the academic year 2009, Naukushu et al., (2012) noted that 30% of the Science Foundation Mathematics students did not pass their Mathematics with 60% or better and in 2010, 32% did not pass their Mathematics with 60% or better. After a critical analysis of the main problem areas, Chirimbana (2013) noted that inverses and compositions of functions are the main areas where the foundation students are performing poorly. Table 1.1 below shows the percentage of compositions by year and the students' overall pass rates, specifically on functions for the 2009-2013 academic years.

Table 1.1: FP Students' performances on inverses and composition functions for the 2009-2013 examinations.

Year	Inverses and compositions of functions Content percentage	Percentage Pass rate On inverses and compositions Of functions only
2009	16	19.9%
2010	14	23.8%
2011	20	16.8%
2012	18	28.7%
2013	15	24.7%

Table 1.1 shows that the performance of SFP students on inverses and compositions of functions for 2009-2013 has always been below 30%, with 2011 recording the minimum of 16.8% and 2012 recording a pass rate of 28.7%. In another study where students were setting academic and behavioural goals in the teaching and learning of mathematics as a way of improving their performances in mathematics (Chirimbana, 2013) found that foundation students performed well in concepts such as sets, matrices, vectors and geometry where the averages were above 60%. One of the reasons which were cited by Naukushu and Chirimbana (2012) about this poor performance on functions was that it is because of the abstract nature in which the concepts on inverses and compositions of functions are presented at the FP. Realising how important functions (inverses and composition) are in setting a base for pre-calculus at first year level, the researcher conjectured that the use of the PBL approach may help not only to improve FP students' performance in mathematics, but also to produce lifelong and autonomous students who are able to apply their acquired knowledge in solving real-life problems.

1.4. PURPOSE OF THE STUDY

Educational approaches that address the deficiencies in modern day educational systems are needed if tertiary institutions are to produce graduates who are not only self-directed and lifelong learners but also mathematically competent to meet the professional responsibilities and demands of the 21st century. As a way of responding to these emerging competence needs, many academic institutions are turning to PBL. Unfortunately, there is a limited amount of systematic research into what constitutes an effective PBL environment (Miflin, 2004). The rationale of this study can be condensed into a simple argument: the topic of functions forms the basis of most the secondary school and university mathematics. Most high school teachers in Namibian High Schools and SFP lecturers who teach this topic find it to be very abstract and requiring more than just procedural knowledge. At this level, the researcher believes that what we teach and how we teach are two important aspects in in teaching. This is specifically because inverses of and composition of functions are subtopics of the topic of functions and should be taught with sensitivity, expertise and skill if teachers are to lay a firm foundation for students' analytical skills in the future (Dolmans et al., 2005).

High school textbooks and the Science Foundation Module for mathematics do not adequately explain, clarify and elaborate this topic adequately, thereby making it difficult for students to understand and comprehend the abstract concepts of inverses and composition of functions. PBL is relatively new in many African classroom contexts and its application does not seem to be well understood. Therefore this study seeks to make a contribution to a better understanding of implementing the PBL approach in an African educational setting.

The present study which focused on the composition of functions, though not directed towards PBL was done by (Lucas, 2003). This study focused on teachers' experiences with composition and inverses of functions and sought to explore the effects of PBL on students' performance in an African academic set up. The overall purpose of the present research was to create a balanced theoretical and empirical report, within the framework of existing PBL learning models directed towards the teaching of compositions and inverses of function.

1.5. RESEARCH QUESTIONS OF THE STUDY

The issues and insights that have been discussed so far lead me to propose the following main research question for this study:

What is the effect of the Problem Based Learning approach on SFP students' understanding of composition and inverses of functions?

In order to answer the question above, the study sought answers to the following sub-questions:

a) What are the SFP students' perceptions with regard to the relevance of inverses and compositions of functions as concepts in a topic that determines their academic destinations?
b) What are the students' preferences of the presentation format for implementing the PBL approach for increased student learning outcomes in the teaching of inverses and composition of functions?

c) How do SFP students experience the PBL approach in the teaching and learning of inverses and composition of functions compared to those who are taught using the traditional lecture method?
d) How do the SFP students' performances on inverses and composition of functions as a result of their PBL experience compare with those of SFP students taught using the predominantly Lecture method at FP?

1.6. ASSUMPTIONS OF THE STUDY

Assumptions are factors which the researcher believes to be true, though these factors are not confirmed to be true. Assumptions add risk to a project like this one since it is possible that they may turn out to be false. This study assumes that all predicaments that may erupt during the course of the study will be manageable and will not deter its completion. The study also operated on the premise that the data collection procedures will be completed within the scheduled six months allowed by the University of Namibia Oshakati Campus Management. It was also assumed that the available PBL resources were sufficient to complete the study and that at least 75 participants out of the 80 who were going to take part in the study were going to endure until the end of the project.

1.7. DELIMITATIONS OF THE STUDY

As outlined in the earlier sections (see 1.4) the purpose of this study was to determine the effect of the PBL approach on the performance of the FP students at the University of Namibia. Therefore the study was limited to the 146 registered students in the Science Bridging course for undergraduate mathematics in the 2013 academic year at the University of Namibia at Oshakati Campus.

1.8. DEFINITION OF KEY TERMS

Problem Based Learning (PBL): This is a student-centred pedagogy in which students learn about a subject through the experience of problem solving.

Lecture Method: This is a teaching method in which the presenter or the instructor teaches orally to a group of class participation.

Foundation Programme (FP): A one year bridging course for undergraduate mathematics.

A Function: is a rule that assigns to each element in a set A exactly one element, called f(x), in a set B. Set A is called the domain and set B is called the range (co-domain) of f.

Inverse function: This is a function that "reverses" another function: if the function *f* applied to an input *x* gives a result of *y*, then applying its inverse *g* to *y* gives the result *x* and vice versa.

Composition function: Given two functions f and g, the composition function $f_o g(x)$ is defined by $(f_o g)(x)=f(g(x))$.The domain of $f_o g$ is the set of all x in the domain of g such that g(x) is in the domain f.

One-to-one function: A function with domain A is said to be **one-to-one** if no two elements of A have the same image; that is $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.

Disadvantaged students: These are students from rural, poorly resourced schools whose academic environments could have made them fail to raise enough grade 12 points to enrol at the university, and are then enrolled in the FP for one year for them gain university access after successfully completing their one year FP course.

Lecturer/teacher: This is the person responsible for delivering lessons in the foundation programme (NB: for this purpose of this study the word teacher and lecturer will be used interchangeably).

1.9. THESIS OUTLINE

This section gives an overview of the remaining chapters in this thesis. Chapter 2 will review literature on PBL. It will then give detailed explanations of the various definitions and approaches to PBL suitable for FP, the various features of PBL, its advantages and disadvantages. It will also address the general implementation, assessment and organization processes of PBL; the models and PBL approaches will also be explained and literature will be used to clarify the research focus. A detailed clarification of Barrows Seven Jump and Shoestring PBL model which were adopted in this study will be provided in this chapter.

Chapter 3 will review literature on the teaching and learning of functions and their relevance to everyday life and their applications to mathematics. It will also explore the different types of functions, including composition and inverses of functions. Different frameworks used in FP to understand functions will also be explored in this chapter. In addition, literature will also be reviewed to clarify the research focus. This chapter will also give a brief overview of the Hypothetical Learning Trajectory (HTL) used in this study. Finally this chapter will present a conceptual framework binding together the PBL models adopted in the study and the frameworks for understanding functions which were adopted in the study.

Chapter 4 will explore the methodological approaches used in this study, including the research design(s), methods, sampling procedures and data collection instruments, the justification thereof, strength and weaknesses. This chapter will also explain justify and justify the data analysis procedure adopted. Research setting and timing will be explained. This chapter will also address research ethics, validity and reliability concerns

Chapter 5 will present and interpret/analyse the data from the various data collection procedures of the research study on the implementation of a PBL approach in the teaching of inverses and compositions of functions. In this chapter special attention will be made on specific research questions to be answered in the study.

Chapter 6 will summarise the main findings and align them to the main aims, objectives and purpose of the study. It will also give suggestions on the areas for further research, the research gap which this study intended to fulfil and recommendations.

1.10 SUMMARY

This chapter gave the background of the study, followed by an overview of the Namibian education. This chapter also gave an overview of the history of FP together with the teaching of mathematics at the SFP at UNAM, Oshakati campus. The chapter also presented the significance and purpose of the study, statement of the problem, purpose and research questions of the study, assumptions, limitations and delimitations of the study and finally the overview of this thesis.

CHAPTER TWO

LITERATURE REVIEW: PROBLEM-BASED LEARNING

2.1. INTRODUCTION

This Chapter will review literature on the Problem Based Learning (PBL) approach, its application and relevance in a wide range of tertiary educational settings. It will look at the origins and the main proponents of the PBL teaching approach and further explore the various models of the PBL approach will be explored. It will explore the role of the PBL approach in the teaching of mathematics. Apart from that, literature will also be used to explore the role of the PBL approach in foundation teaching, its implementation, evaluation, assessment, curriculum design implications, aims, criticisms, students' and teachers' roles, its advantages and disadvantages. In addition to that, the researcher will use literature to explore factors on PBL curriculum design, which will include the nature of tutorials and assessment. In addition, this chapter will explore the philosophical principles underlying the PBL approach will also be reviewed using available literature. Furthermore, literature will be used to establish a link between the PBL approach and Realistic Mathematics Education (RME). Finally clarification of the research focus will be then be done using literature.

2.2. DEFINITIONS AND CHARACTERISTICS OF PBL

2.2.1. Definitions of PBL

PBL is a learner-centred and enquiry-based approach which posits that more effective learning emanates from the process of working towards the understanding and resolution of a problem (Barrows, 1996). Barret (2004), on the other hand, defines PBL as the training institutions' response to the incessantly changing needs of the workplaces. Therefore, PBL aims at creating a firm and concrete knowledge base which students can always utilise later in life.

Azer (2005) defines problem-based learning (PBL) as a student-centred instructional strategy in which students collaboratively solve problems and reflect on their experiences. PBL is a teaching approach in which the content is introduced through the process of problem solving, rather than introducing problems after teaching the content

(Jervis and Jervis, 2006). Saarinen-Rahiika, Binkley and Hayes (2008) define PBL as an instructional method that challenges students to "learn to learn," working cooperatively in groups to seek solutions to real world problems. According to Saarinen-Rahiika et al. (2008), these problems are used to engage students' curiosity and to initiate the learning of the subject matter. Tiwari (2008) defines the PBL as a learner-centred instructional (and curricular) approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem.

From all the various definitions of PBL given above, it can be concluded that the PBL approach uses real-life problems, and engages students in solving a problem in a learning environment. From the researcher’s own perspective, Problem Based Learning is a learner-centred and enquiry based educational approach which works on the assumption that more meaningful and effective learning comes from the process of working towards the understanding and resolution of a problem.

Based on these definitions, it can be argued that the teaching that takes place under PBL starts with the presentation of a problem where students have to work through under the guidance of the lecturer.

2.2.2. Characteristics of PBL.

PBL does not only accord learners an opportunity to solve problems, but also allows them to identify the problems themselves using real-life situations. Therefore PBL is conducive to FP where the accrued FP knowledge will continue to be utilised at the university. Dolmans et al. (2005) state that students work in small groups of between five and eight students in solving problems. In his theory of learning, Freire (1972) is of the view that students must construct their knowledge based on the prior knowledge they possess and teachers must learn how students understand the world to understand how students can learn. Freire further argue that once teachers understand how students learn, they are also in a position to elicit what students’ previous experiences are for them to create relevent real-life teaching material that will benefit the students.

PBL has the advantage that it provides learners who have limited knowledge of a field of study with an opportunity to achieve high levels of cognitive engagement (Kirschner et al., 2006). Since the students' groups which are used in PBL enable the weaker students to learn from the stronger ones, this collaboration among students will help them to improve their performances. The process of solving problems allows learners to discover or develop new ways to solve new real-world problems, which is a skill needed by learners when they are exposed to new educational or work environments. Therefore PBL is an approach that challenges students to learn through engagement in a real problem (Dubinsky, Carlson, Hawks, Nichols, & Harel, 2004). This will enable the entire learning environment to be rich and students will benefit more through their real world experiences with the problems being solved.

One of the characteristics of PBL is that it uses real life problems in small groups. When people confront a problem, they analyse, identify what the problem is, make enquiries of the information they need to know about the problem and come up with a hypothesis and solutions. One main characteristics of PBL as asserted by Downing (2001) is that it achieves instructional goals such as promoting students' knowledge transfer. This includes making use of knowledge in context, helping students develop reasoning skills, self-directed learning skills, and increasing the motivation for learning among the students. In an educational set up, PBL allegedly has the power to create a problem-anchored learning environment to take up this natural enquiry process to pursue and use knowledge effectively (Albanese and Mitchell, 2004). Therefore it can be argued that PBL fits well with progressive learner-centred views emphasising direct experience and individual enquiry.

Moreover, PBL claims capability to provide graduates with the outstanding ability of managing academic and professional problems of those who seek their services in a competent manner (Mergel, 2008). Barrows (1996) also states that the emergence of PBL is as a response to the changing needs in the development of integrated knowledge bases, problem solving skills, effective self-directed learning skills and team skills. Thus PBL appears to have the potential to provide a learning environment with an emphasis on problem-initiated, self-directed and collaborative learning.

The main characteristics of PBL according to Barrows (1996) are (1) Learning is driven by challenging, open-ended, ill-defined and ill-structured problems (2) Students generally work in collaborative groups (3) Teachers take on the role of "facilitators" of learning. PBL positions students in simulated real world, working and professional contexts which involve policy, process, and ethical problems that will need to be understood and resolved. In PBL, students are encouraged to take responsibility for their group and organize and direct the learning process with support from a tutor or instructor. Barrows (1996) claims that PBL can be used to enhance content knowledge and foster the development of communication, problem-solving, and self-directed learning skills. Barret (2004) argues that by working through a combination of learning strategies to discover the nature of a problem, understanding the constraints and options to its resolution, defining the input variables, and understanding the viewpoints involved, students learn to negotiate the complex sociological nature of the problem and how competing resolutions may inform decision-making.

Lohman (2004) argues that in PBL environments, students act as professionals and confront problems as they occur - with fuzzy edges, insufficient information, and a need to determine the best solution possible by a given date. This is the manner in which engineers, doctors, and even teachers, approach problem solving, unlike many classrooms where teachers are the "sage on the stage" and guide students to neat solutions to contrived problems.

2.3. OVERVIEW OF THE HISTORICAL DEVELOPMENT OF PBL

2.3.1. The origin of PBL

The origin of Problem-Based Learning goes back to 1920 (Yuruker, 2011). Celestin Freinet, a primary school teacher, came back injured from World War I. He saw himself incapable of speaking and teaching in front of a class for extended periods of time. His injuries forced him to seek a new methodology that would allow him to continue his professional activities in a satisfactory way. He established a system, in which the pupils played an active role in learning. The mainstays of this approach were communications skills, cooperative learning, self-responsibility and self-evaluation of their learning process: all these are features of PBL. Graaf and Kolmos (2003) consider modern PBL to be a teaching approach that originated in Canada at McMaster Medical University in the late 1960s. The main proponent of PBL, according to Barret (2004) was Barrows Haward. This teaching approach was first applied to some medical students at McMaster University but later spread to three other Universities; University of Limburg at Maastricht (the Netherlands), University of Newcastle (Australia) and the University of New Mexico (United States of America) (Nguyen, 2009). These Universities took over the PBL model from McMaster University, but later on various adaptations were made and the model soon found its way to various academic and professional disciplines - business, education, dentistry, health sciences, law, engineering (Nguyen, 2009).

The PBL approach was used to enhance the quality of medical education by moving from a subject and lecture-based curriculum to an integrated curriculum which is structured through real-life problems which cross traditional boundaries (Nguyen, 2009). This teaching approach is deeply rooted in the “project work” of William Kilpatrick (Kain, 2003). According to Kilpatrick, learners should not be provided with answers but rather with proper practical experiences in learning to help them create the questions and seek answers and solutions to those questions and problems.

2.3.2. Problem-based learning as an (innovative) approach in a foundation programme

Students’ performance in the SFP relies on the methods or approaches the teacher uses, and the study habits of the student (Richardson, 2005). Richardson further argues that there are three approaches to studying in foundation and higher education which students have to utilise in order to improve their performances. There is the deep approach, which is based on understanding the course material; the surface approach, based on memorising the course materials for the purpose of assessment, and a strategic approach, based upon obtaining the highest grade (Dahlgren, 2000). However, a student can exhibit all the three studying approaches in different situations. The choice of a study approach depends upon the content to be taught, the context in which the teaching is taking place and the demands of a particular task in a particular subject (Ramsden, 2009). Teaching and learning at the bridging course for undergraduate mathematics does not end there, but lead to tertiary education. Therefore the teaching approach that best suits foundation learning is the deep approach which is best supported by PBL.

Richardson (2005) conducted a study by comparing PBL and the traditional, subject–based curricular. The results showed that students following the problem-based curricular are more likely to adopt a deep approach and they are less likely to adopt a surface approach to studying. Interventions aimed at inducing desirable approaches proved to be in ineffective. Marton (2006) explains why students adopt different approaches on the same course. He states that students who adopt a deep approach integrated in a PBL approach to studying take an active role and see learning as something that they have to do. They have responsibility over their own learning, and they are motivated, whereas those students who adopt a surface approach through the lecture method or any other traditional teacher-based methods take a passive role in the learning process and perceive learning as something that just happens to them, with their little or no involvement at all. Basing on what teaching methods are adopted by teachers, students perceive education differently. Sweden (2006, p. 19) found five different conceptions about the concept "learning" among British foundation students :(i) Learning as an increase in knowledge (ii) Learning as memorisation (iii) Learning as the acquisition of facts or procedures (iv) Learning as an abstraction of meaning (v) Learning as an interpretative process aimed at the understanding of reality. The way foundation students will perceive learning will also depends on how they are talk in terms of approaches and teaching methods. Therefore students who are taught using the PBL approach are likely to perceive learning as an interpretative process aimed at the understanding of reality.

Aley (2006) found that FP lecturers use a variety of teaching approaches some of the which are teacher-focused, aimed at transmitting information to students; while others are student-focused and aimed at bringing about conceptual change in students. Students whose teachers adopt a student-focused approach to teaching are more likely to adopt a deep approach to learning and less likely to adopt a surface approach to learning than students whose teachers adopt a teacher-focused approach (Trignell & Prosser, 2006). In other words, student-focused teaching approaches like PBL have the potential to engender more desirable study habits and metacognitive skills in the students than does a teacher-focused approach. Trignell and Prosser further posit that students who are taught using the PBL approach are more likely to value their teaching and learning. However, they advise on keeping class sizes as small as possible so that both the teacher and the students have total control over the learning that transpires in the classroom. In other words, if PBL is to be implemented successfully in FPs, students' learning groups should be kept as small as possible to make it easier for the facilitator to reach out to all the groups.

2.4. PHILOSOPHICAL PRINCIPLES UNDERPINNING PROBLEM-BASED LEARNING AND FP

2.4.1. Philosophical Principles underpinning PBL

Zieber asserts that all philosophical underpinnings that are now associated with PBL can be said to be *ex post facto,* since they came later to explain an academic finding that started without them. Zieber (2006) argues that PBL is closely associated with progressivism, constructivism and humanist philosophies. Its philosophical roots though, can be traced back to Socrates, who utilized problems he faced with his students so that through their questions, he could help them to explore "their assumptions, their values and inadequacies of their preferred solutions" (Savin-Baden and Major, 2004, p. 4). Furthermore, a mention can be made of Aristotle as well, who made the suggestion that students begin problem-solving by determining their perceptions and beliefs. Models of learning in ancient times also promoted learning-by-doing as core values for students of those times (Rideout, 2005).

Rideout further states that the hallmark of progressive education is its emphasis on vocational and manual education, practical learning, scientific enquiry, community involvement and responsiveness to social problems. Pragmatic educational philosophies are subsets of progressivism. Pragmatism assumes the centrality of human experience, which is placed at all ways of arriving at knowledge (Severiens and Schmidt, 2008). There is an emphasis on consequences of actions to determine whether they are true or good and an emphasis on social reformation. Zieber (2006) contends that Dewey promoted the notion that the:

> "Highest ideal of the progressive movement was education for democracy" which Dewey defined as "people engaged in joint activities to solve their common problems"(p. 3).

The existence of these inseparable individuals and social goals in education and Dewey's belief that the mind and its formation was a communal process, and that the individual and his or her society had no meaning apart from each other are reflected in PBL process group work, community and collaboration which form the basis of its social context of learning (Azer, 2004; Tan, 2004; Kirschner et al., 2006 & Rideout, 2005). Therefore, PBL is instrumental in achieving social goals in the field of education.

Basing on this argument, one can clearly state that Dewey adopted a constructivist perspective of teaching and learning, which shifted from the teacher as the controller to that of being an intellectual leader. The instructor's responsibility is to organize, stimulate, instigate and evaluate the highly complex educational processes, and provide a context that promotes active learning among the students (Schmidt and Norman, 2003). Dewey's constructivist philosophy fits well with PBL in that, PBL advocates for, and promotes student-centred learning by requiring students to be active in the process of collaboration, decision-making, and pursuing knowledge through a variety of sources, while the teacher takes the role of a facilitator (Barrows, 1996). It is for this reason that various educational institutions are making a paradigm shift to PBL because of these lifelong benefits associated with the approach.

Dewey viewed learning as the process of "finding out" (Savin-Baden and Major, 2004). He asserted that true learning is established through discovery, guided by mentoring, and that learning was not a direct outcome of knowledge transmission (Zieber, 2006). All these notions support the PBL learning approach, which combines the process of learning and content as equal aspects in learning (Azer, 2005). Based on these arguments, it can be claimed that PBL seeks to address Dewey's activity-based approach to learning, which posits that when students encounter a novel situation, a state of disequilibrium is created in them which provides the incentive for real learning to take place.

The constructivist philosophy is concerned with how we come to understand what we know (Zieber, 2006). Constructivists believe that knowledge is not constructed in a vacuum, but instead prior knowledge is needed to construct knew knowledge (Conoldi, 2008). This principle of prior knowledge, being central in constructing new knowledge, is the backbone of the PBL approach as foundation mathematics students need to have an understanding of basic algebra, algebraic manipulations and subject of formula before they are exposed to composition and inverses of functions. In PBL, students spend time solving real-life problems which are applied to a specific subject.

The experiences that they gain in doing this eventually help them to solve real-world problems. In a PBL environment learning becomes relevant and meaningful when it is applied to real world problems and that new knowledge is related to what is already known. Furthermore, Jervis and Jervis (2006) state that the underlying principle is that knowledge is constructed through social negotiation with the environment. PBL also resonates with the social constructivist theory of Vygotsky and others. It is related to social-cultural and constructivist theories of learning and instructional design. Early constructivist philosophy was focused on the learner-centred approach, and he argued that learners were not passive knowledge recipients but were actively involved in the knowledge construction process (Loyens and Gijbels, 2008). Support systems which include resources germane to the problem domain as well as instructional staff are provided to scaffold students' skills "just in time" and within their learning comfort zone (Vygotsky's Zone of Proximal Development) (Vygotsky, 1978).

Overall, constructivism gave rise to instructional design principles that contribute to the PBL environment (Zieber, 2006). According to Savery and Duffy (2006) these principles are: (1) anchoring the activities to a larger purpose or problem (2) support the learner in developing total ownership of the problem(3) design an authentic task which the learner can engage in (4) design the problem and the environment in such a way that they reflect real-world complexities (5) give the learner total ownership of the process of developing solutions to the designed problems(6) design the environment to support and challenge the learners' thinking capacity(7) promote the habit of testing of ideas against alternative views(8) make a promotion of the reflection on the old content and processes. The teaching and learning of compositions and inverses of functions in a bridging course for undergraduate mathematics requires students' prior knowledge of equations and change of subject of formula for them to be able to construct the relevant knowledge they need during the PBL experiences.

2.4.2. Theoretical frameworks underpinning FP

Teaching and learning in foundation (access) programmes has been influenced by a number of teaching philosophies, ranging from behaviourist to situated cognition (Kloot et al., 2008). Rollnick and Holtman (2010) argue that given the students' social, economic situation it would not be sufficient to only consider the theoretical frameworks that only consider the students' cognitive processes. These authors further argue that more accommodating theoretical frameworks for FP teaching and learning should include the sociological approaches which include focus on "identity development", "community of practice" and the understanding of "ground rules" of the discipline. When students are admitted into the SFP, they gain epistemological access to the process of knowledge construction in their discipline (Mabila et al., 2006). The researcher argues that this has to be the "new" goal of higher education in mathematics and sciences if students have to succeed and that this is only possible through reconceptualising what we recognise as teaching and how foundation students view learning. This places greater demands on academic staff, students and the teaching and learning environment and calls for a broader theoretical lens through which to view foundation students and their learning experiences (Case, 2008).

On the other hand Kloot et al.(2008) identify three different models underpinning FP learning. These authors argue that FP learning is a "more time more tuition" form of learning.They further argue though this may not be a theoretical basis, a tenet of the FP learning is that disadvantaged students need *more time* and should be provided with *more tuition* if they are to succeed at the University. SFP at Oshakati receive tutorials in mathematics inorder to meet the goal of "*more time more tuition".* Kloot et al. (2008) identify the basic skills model,the integrated skills model and the holistic model as models that guide the teaching and learning that take place in a FP. According to these authors these three models are inclusive of all the socio-economic backgrounds of the foundation students.

2.3.2.1.The basic skills model

In the basic skills model in a FP is seen as attempting to equip students with certain academic skills.Therefore the basic skills model best suites this phenomenon of acquiring these most needed skills.It is envisaged that foundation students were “disadvantaged” in that they lacked certain skills. Based on this, the focus of FP is on developing students in terms of improving their cognitive, communication and study skills (Kloot et al., 2008).

Since most students enrolling in the FP are from disadvantaged backgrounds and can barely construct a grammatically correct English sentence since English is their second language, English communication skills are offered to improve their writing, reading and communication skills (Holtman., 2004). In this regard, theories of language are essential in meeting this goal. It is understood that in their previous schools, foundation students were taught English using “rehearsing and drilling grammar…and copying and correcting “faulty” work” (Boughey, 2003, p. 10). Therefore bridging course for undergraduate mathematics teaching at Oshakati campus is focused on teaching practical skills and life skills to try and help students overcome their under-preparedness for them to join the tertiary education programmes

2.3.2.2.The integrated skills model

As highlighted earlier the emergence of the SFP may be regarded as a manifestation of a change in attitude on the part of the universities as they began to take ownership of the problem of educational “disadvantage” (Kloot et al., 2008)).This was evident in the universities’ shift away from academic support towards academic development which enabled more integrated forms of assistance and accommodation for disadvantaged foundation students such as the ones at Oshakati Campus.

In terms of teaching and learning FP practitioners realised that skills taught in isolation did not transfer effectively across subject areas, and that students’ tendency to rote learn could not be dealt with by a quick skills study course (Miranda, Nakashole, &

Chirimbana, 2013). Neither could their difficulties with English be resolved by the Namibian College for Open Learning (NAMCOL) English Communication module nor could their lack of number sense abilities be resolved by NAMCOL. It can be concluded that the approach of trying to fix the surface forms of language such as grammar and sentence construction and improvement of students' basic mathematical abilities, was not adequate in dealing with deep seated consequences of poor high school education. The design of the SFP mathematics curriculum was therefore intended to identify and explicate the skills and processes required for engagement with subsequent years of a mainstream degree and then integrating these skills with first year content (Kloot et al., 2008). Kloot et al. (2008) further state that the FP course materials were designed to encourage students to develop a valid conceptual framework rather than simply redefining a rote learning approach. Therefore the integrated model of bridging course for undergraduate mathematics is in line with the purpose of mainstreaming successful foundation students in the university first year. With regard to this model, SFP students from Oshakati campus have enrolled in various sciences related degrees in Namibia for science related degrees. Table 2.1. below shows the FP success rate since 2005-2011.

Table 2. 2: Enrolment success rate for students completing the FP at Oshakati Campus 2005-2013: Adapted from (UNAM, 2014).

Year	Number of students enrolled	Failed to pass FP	Failed to pass Mathematics	Education	Medical field	Other fields
2005	50	4	7	60%	20%	16%
2006	62	5	7	78%	15%	6%
2007	83	7	8	67%	25%	5%
2008	85	6	10	79%	18%	4%
2009	119	8	13	69%	24%	7%
2010	127	9	10	57%	32%	9%
2011	137	10	12	77%	14%	5%
2012	145	15	15	68%	26%	6%
2013	146	16	19	70%	16%	8%

2.3.2.3.The Holistic skills model

In the Holistic skills model the SFP mathematics curriculum is designed in such a way that a student who successfully completes the Programme would have acquired the necessary skills not only to excel academically, but also to manage time and other resources needed in the learning processes and life in general (Kloot et al., 2008). These students are well equipped to manage stressful environments and can cope well in any situation that may be strenuous to them since the course has some life skills management aspects that are mainstreamed in the study course (UNAM, 2005). SFP learning is of special relevance for students who were previously exposed to inferior schooling since it means that lecturers need to take into account students' prior knowledge and life experiences and build on them. The students in foundation programmes have to construct their own knowledge based on their "rough" previous experiences. In this current study the researcher classifies SFP learning under Vygotsky (1978) social constructivism to learning as highlighted earlier. Therefore the holistic model of foundation programs in more inclusive with regard to the nature of the students, the nature of learning that is expected to transpire in the FP and the expected type of product that comes out of the FP (Kirschner et al., 2006). Foundation program students can be said to be producing life-long learners since whatever they learn in the FP has to be applied in their future academic and professional endeavors.

2.5. LEARNING THEORIES IN A FP AND THEIR CONNECTION TO PBL

The main goal of Science Bridging course for undergraduate mathematics (SFP) at the University of Namibia is to promote learning among students and increase the numbers of university graduates in science related qualifications in tertiary institutions (Naukushu et al., 2012). Every instruction that is done at FP should promote learning that will ultimately lead to the fulfilment of this overall goal. The study materials are also designed in a way that correlates with the method of instruction used at the FP. The mode of delivery and material design are the main determining factors of students' success in a FP (Rovai, 2002). The learning theories that are relevant in the Namibian education system are also relevant to the form of learning that takes place in a FP. Consequently, constructivist, cognitivists and behaviourists learning theories are also relevant to the learning that takes place in a FP, and the fundamental acquisition of knowledge (what) is best suited to traditional teaching and behaviourist learning strategies, whereas more advanced learning (how) is more inclined towards constructivists' learning environments which make use of real life, personal applications and contextual learning.

Mergel (2008) states that behaviourism is a learning theory that is concerned with observable changes in behaviour. Behavioural patterns are continuously repeated by the learner until they become automatic to the learner. Anderson and Elloumi (2004) argue that behaviourism works with the notion that learning is a repetition of experiences. Rote learning and drilling which are normally used in exam oriented classrooms do not allow students to extend their understanding of an idea. Learning is said to have occurred if, in response to the teaching and learning activity there are observable traits of behavioural change (Kwan and Ko, 2009). Most of the teaching that takes place at the science foundation was based on the behaviourist learning perspective. Good and Skeates (2006) contend that this learning strategy has been criticised for the fact that it ignores the thought processes that take place in the learner's mind, which might not be reflected in a behavioural change.

The cognitivist learning theory is mainly concerned with the thought processes that occur in the learner's mind and that underpin the behaviour (De Beer, 2006). Marchant (2006) urges that cognitivists view learning in terms of the internal cognitive knowledge structures and processes of the mind that the learners use to store information. Cognitive psychology views of learning from the point that learners use different types of memories for personal information processing. This theory has shown that the art of learning is an internal process that involves the use of memory and thinking, and that reflection plays a significant role in the process of learning (Gibbings, 2008). Based on this it should be stated that the focus of learning should not only be on external behaviour, but also in PBL for FP students.

Another learning theory that can be used to explain learning that takes place in a FP is constructivism. The origin of constructivism can be traced back to as early as 1932 with Bartlett as the main proponent (Bartlett, 2005). Gibbings (2008) claims that the constructivist theory was founded by Dewey (1938); later refined by Piaget (1952) and later on modified by Vygotsky (1978), and Glasserfield (1995). As already indicated earlier, constructivism works on the premise that an individual's knowledge is developed on the basis of their past experiences, internal cognitive structures as well as underlying beliefs. Growth in conceptual understanding comes from changing these internal representations which come as a result of sharing of perspectives and through collaborative learning. This is one of the greatest strength of constructivism, since the learner is able to make an interpretation of realities from multiple perspectives and is better prepared to handle real life problems and other novel scenarios (Vygotsky, 1978). Fatade et al. (2013) assert that constructivism creates an opportunity for individuals to share their individual reality if it is applied in a team based learning environment. Basing on these arguments, it is therefore justifiable to claim that constructivism fits well with the PBL approach in a FP since it advocates a more open-ended learning experience where the learning outcomes may not necessarily be the same for each learner.

Constructivists believe that learning does not take place in a vacuum but prior knowledge is needed to scaffold learners to higher level knowledge (Bogdan, 2000). In this learning theory, Biggs (2009) believes that knowledge is constructed individually by the learner and that a deeper level of information processing results in long term memory since this involves the creation of linkages between existing and new information in the mind. Kirschner et al. (2006) argue that this will ultimately lead to greater information retention. This makes the recognition of both the positive and negative effects that prior learning and experience have on the learning of new material. Other researchers have consequently argued that SFP educators should make provision for a variety of teaching and learning strategies for students, which will accommodate individual learning differences among the learners (Naukushu et al., 2012; Holtman et al., 2004; Rollnick and Holtman, 2010; Kloot et al., 2008). This observation has led to a plethora of research on different teaching methods in which PBL, the central aim of this study, is one of them. Gibbings (2008) asserts that constructivism builds on behaviourism and cognitivist theories in the sense that it accepts multiple perspectives in as much as learners construct their own personal reality of the world based on their own individual as well as collaborative experiences. Anderson and Elloumi (2004) contend that PBL is a holistic teaching approach that meets both the academic and professional needs of the bridging course for undergraduate mathematics of the 21st century. Loyens and Gijbels (2008) constructivist learning situations should resemble real-life situations as in PBL.Therefore the implementation of PBL at Oshakati FP is to try and meet the modern workplace demands as advocated by modern educationists.

Learners construct their individual knowledge and that enables them to make the information meaningful to themselves in their own personal contexts (situated learning or situated cognition), which is a key emphasis of constructivism (Kluwe, 2007). Barlett (2005) suggests that in order to fully promote situated learning in constructivism, opportunities must be created for students to develop personal knowledge and create personal and individual meaning. This can be done by creating an opportunity for students to contextualise and apply whatever they have learnt into real-life situations PBL accords students such an opportunity since it deals with real-life problems.

Students in the SFP are in the process of undergoing academic transformation; their entry into tertiary education depends on the extent to which their academic transformation has taken place. For this reason, “transformative learning” which can arguably be said to be a subset of psychology and constructivism, is also relevant to the foundation education (Tynjala, 1999; Biggs, 2009; Rollnick and Holtman, 2010). When students encounter a concept that tends to challenge their existing knowledge, they take time to reflect on it. In the process of reflecting on the new idea, they may decide whether the new idea can be related to pre-concept that already existed in them, or may decide to reject the new idea completely, or they may make an adjustment of their existing knowledge in order to try and accommodate the newly experienced concept (Piaget, 1978). If they opt to accommodate the new idea, then one can successfully claim that they have constructed new knowledge. When this has taken place, Rovai (2002), states that they have undergone a transformation since their original perspective has been “transformed” to a new understanding.

The researcher can boldly claim that constructivism is a theory of learning and knowing, and was originally embedded in the realm of developmental psychology and cognitivist related to pedagogy. On the other hand, the transformative theory is embedded in the realm of foundation learning and one can claim that its emergence is from the Freireian (Freire, 1972) concept of “conscientisation” and learning for emancipation and social reconstruction. However, Gibbings (2008) is of the view that the pre-requisite for transformative learning in a PBL approach is the degree of learner maturity, their ability to critically apply their classroom knowledge to solve real-life problems. Gibbings (2008) considers transformative learning to be more viable when the students’ maturity has passed the concrete learning stage, and they must have developed sufficient skills to become independent learners who can learn with minimum supervision. PBL can also be utilised for the same purpose of achieving transformative learning among FP students.

The constructivist and transformative theories are two different theories of learning though they have a link between themselves (Tynjala, 1999; Loyens and Gijbels, 2008). The theory of constructivism focuses more on the explanation of the learning process, from the beginning to the end, whereas the transformation theory focuses on one particular learning theory which is transformative learning. Tynjala (1999) argues that the occurrence of transformative learning rests, not only on the learners’ experience of constructivist learning, but an experience of the transformation of their perspectives of the subject. In this regard, transformative learning is constructivist in its epistemological sequencing, but constructivism is not necessarily transformative. Learning can occur by simply adding together knowledge or learning new skills, and this learning can involve learning schemas in one’s own mind as well. Students, including those in an FP, can interpret new information using knowledge that they have already acquired. They activate prior knowledge and try to relate new information to the knowledge they already possess (Loyens and Gijbels, 2008).

2.6. VARIETIES OF PBL APPROACHES APPLICABLE TO TERTIARY EDUCATION.

PBL is an inductive teaching approach that exists in various forms such as inquiry-based learning, case study based learning, project-based learning, just-in-time teaching, Problem-Based Learning, discovery learning and case-based teaching (Prince and Felder, 2007). The role of inductive teaching is that the teacher starts with an “ill-structured problem” in the case of case-based learning and asks the students to analyse and find a solution. The learners have to work in collaborative groups to find solutions to problems through searching facts, skills and conceptual understanding under the guidance and facilitation of the teacher. What is common about inductive teaching methods according to Nguyen (2009) is that the learners are exposed to the challenge first, where they have to make their own intuitive thinking and investigations to find an amicable solution to the problem at hand. Table 2.3 shows the various characteristics of inductive teaching methods.

Table .2.3: Characteristics of different inductive teaching approaches (adopted from (Savin-Baden & Major, 2004)

Teaching method	**General description of its nature**
Problem-Based learning	Learners are required to apply their knowledge to produce something such as a process or design. The students' application of prior knowledge to complete a product forms the central part of this teaching approach (Prince & Felder, 2007).
Just-on-time learning	Learners answer electronic conceptual questions before the beginning of each lesson. The teacher who takes the responsibility of a facilitator will then work to make corrections of the raised misconceptions in the class. The implementation of this teaching approach according to Prince & Felder (2007) is very demanding since it requires thorough preparation on the side of the teacher/facilitator to prepare the conceptual questions that meet the right standards and a web-based management system is also required that will make a tabulation of the learners' responses which may be reviewed by the teacher/facilitator.
Inquiry-based teaching	Learners are exposed to a scenario which could be a question to be answered, an observation or some data to be interpreted, or a statistical hypothesis to be tested. As learners respond to these questions the process of learning is completed. This method according to Prince & Felder (2007), suits well with

	problems that require experimental investigation or involve materials and situations which learners are already familiar with, and this method fully promotes skill development among the learners.
Discovery learning	Learners are given a problem and are left to find a solution on their own. The facilitator may provide feedback but his effort to help students is keep at a minimum until the learners have done their part. Prince & Felder (2007) state that discovery learning relies mostly on trial and error techniques.
Case-study based teaching	Learners are presented with case studies or scenarios which may be present in their real professional encounters/practices. Problems are solved by learners after first making an examination of their preconception and make adjustments to accommodate case realities. Cases are rich in contextual details, and learners make a utilisation of what is already familiar to them and build on it too and make new discoveries in the learning process (Lohman, 2004).

2.7. RATIONALE FOR PBL IN A SFP LEARNING

2.7.1. The main role of a PBL approach in a FP

PBL approaches have been implemented in Australia, America and Canada over the past four decades in the medical field, engineering etc. Its application in an African educational set-up PBL has been applied in mathematics education by (Fatade, Mogari, & Arigbabu, 2013; Malan et al., 2013). Since this educational approach was first popularised, it has been implemented at Nelson Mandela Metropolitan University School of medicine, the University of KwaZulu Natal School of Undergraduate Medical Education and University of Stellenbosch, all of which are in South Africa (Ramsden, 2009). Studies by Dolman et al. (2008) to determine how students from diverse backgrounds perceive the effectiveness of the process and content of the PBL tutorials among Medical students at Nelson Mandela School of Medicine at the University of KwaZulu Natal revealed that students who learned in a group were more motivated than those who had to do tutorial work individually. This study further revealed that Medical students at the University of KwaZulu Natal, regardless of their social backgrounds, had a positive perception of small learning.

Velman, DeWet, Mokhele & Bouwer (2010), after reviewing the twelve National Critical Cross-Field and Developmental Outcomes (COs) which are provided to all the institutions of higher learning in South Africa, observed that PBL is one of those teaching approaches that can be utilised in order to make this national goal in Higher education a reality. PBL has proved to be an effective solution to meet the professional demands of the 21st century as compared to the old school didactic models that do not deliver graduates who meet the modern day industrial demands (Veldman et al., 2010). They further state that the industry itself has changed considerably over the past decades, and as such there is a need for graduates with different attributes compared to the knowledge driven qualities that were required in the past. Therefore, PBL in an African Higher education environment provides an appropriate strategy in addressing these needs (Iputo and Kwizera, 2005).

Furthermore, Iputo and Kwizera (2005) in a study to compare the effectiveness of various teaching approaches among medical students taught between 1989 and 2002, noted that at the University of Transkei Medical School, students who were taught using the PBL which they called the Community Based Education (CBE) had an improved academic performance as compared to those students who were taught using the lecture based, traditional teaching approaches. Students who were trained using the PBL approach were better equipped to face the realities of the work environment than those who were taught using traditional methods. The employment of PBL to promote active learning and the effective use of technology in undergraduate education has become the home of the PBL clearinghouse - a peer-reviewed database of more than 100 field-tested PBL problems across a wide variety of disciplines (Watson, 2002).

Kwan and Ko (2009) argue that students need to be effective problem solvers and should understand how their minds function. In other words, they need to have a perception of how they perform important cognitive tasks such as remembering, learning and problem solving. Therefore PBL appears to have the potential to promote more metacognitive development when compared to non PBL approaches which may not require the same self-reflexive performance (Hacker, 2008). The PBL approach appears to have the potential to create the platform for the student to think about his or her own and others' thought processes, and the thinker can pay attention to and change his or her thinking, a notion called executive control processes. In this connection Bogdan (2000) recommends that cognitive tasks in PBL should be set in a sequential order (remembering the things learned earlier that might help with the current task or a problem solving task) and metacognitive tasks (monitoring and directing the entire process of problem solving), should lay more stress on learning how to learn. If students feel confident that they can solve problems, they tend to do better work (Conoldi, 2008). In a sense, the learners need to use and describe the processes involved in their mental activity.

Foundation students who are taught using the PBL approach will most likely have a clear understanding of ordinary thinking and awareness and understanding of effective thinking strategies (Biggs, 2009). Such students are more likely to succeed in their future academic and professional deliveries in future. Arbak, Bretel and Agrawal (2007) argue that class size, a perception that student-led learning is lacking in discipline, the need for a willing administration and difficulty in getting students and teachers to adapt to becoming facilitators of learning rather than lecturers were the most pressing problems they had to overcome. Therefore in a SFP, successiful adoption of a PBL approach lies upon the lecturer ability to resume the role of being a facilitator rather than a teacher.

Biggs (2009) identifies the aim of FP education as getting students to fully develop the functioning knowledge which allows them to integrate three aspects of knowledge: academic knowledge base (declarative knowledge), skills required for that particular profession (procedural knowledge) and the context for using the skills to solve problems (conditional knowledge). Similarly, Conoldi (2008) argues that PBL by its nature requires a different way of using knowledge to solve everyday problems, and it is this "functioning" knowledge that involves the metacognitive knowledge. Consequently, because of the fact that PBL uses real world problem cases as its starting point, the processes involved in solving varieties of problems leads to a full development of the three characteristics of metacognition defined by Biggs (2009) above.

There are several types of PBL models all rely on the students' ability to successfully monitor and skilfully direct the entire process of problem solving, bringing memory of concepts and processes learned earlier to bear upon the current problem (Conoldi, 2008).The entire learning sequence for PBL and the motivational context of learning is set up by real life problems; learners activated through group, peer and through their interaction with the teacher who takes the role of a facilitator (Eley, 2002). A knowledge base of relevant materials is constructed and applied to deal with the case; and the case is then reviewed, requires reflection upon declarative, procedural, and conditional knowledge (Vermetten et al., 2009). Therefore, PBL should theoretically be able to positively influence rapid development of metacognition among foundation students (Bogdan, 2000). Conoldi (2008) argue that for maximum benefits to be obtained from the PBL learning approach, appropriate levels of scaffolding and support must be provided inorder for students' metacognition capacity to be fully developed.The extent of success of any learning process lies apon the provision of quality learning experience to the students (Colliver, 2001). FP lecturers need pedagogies that can teach their students problem solving skills. PBL specifically engages students in solving large, complex, interdisciplinary problems while emphasizing the need for a deep, conceptual understanding. This will enable FP students to apply their knowledge in a variety of fieds in future and can make them proper professionional assets for the 21st century.

2.7.2. Previous research on PBL effectiveness

The introduction of PBL, which, as already noted earlier, first emerged in medical education and later became popular in other fields has gained great success and recognition more especially in the education fraternity. Kain (2003) notes that PBL has been applied widely in American, Australian and Canadian educational contexts though its application in African educational systems is still very low. Nguyen (2009) points out that PBL was renamed Issue-Based learning (IBL) when it was implemented in Social Studies at the University of New South Wales in Australia Nguyen further states that since the introduction of IBL at the University of New South Wales Social Studies department, great success in university education was reported and the IBL was later imitated by other departments like Engineering, Education, Biochemistry, Physics, and Mathematics. The most commonly used models in tertiary instuitions are the showestring, the funnel or the the Seven Jump approach (Albanese and Mitchell, 2004).

Biggs (2009) reports that fourth year undergraduate nursing programme students who implemented the PBL shoestring approach in Canada showed a great improvement in their level of understanding, comprehension, self-direction and application and demonstration of a holistic understanding of their nursing knowledge. However, research showed that the use of the PBL approach limited leaners' application and use of their psychomotor skills and were also lacking important basics on anatomy and physiology (Gibbings, 2008). Though the PBL implementation enhances student self-direction and autonomy in a FP, some adjustments must be made in the programme to improve all the other necessary skills so that the product is holistically skilled. Therefore, FP should have programmes that are meant to cater for the social needs of the students since they are thought to be coming from disadvantaged communinities.

Studies by Dahlgren (2000) on Swedish University mathematics students investigated three different PBL problems different results on a stated range of objectives. These mathematics first year students were expected to examine the meaning of some pre-listed objectives. The study showed that the funnel PBL approach allowed students to make a meaning out of the pre-listed objectives. Fatade et al. (2013) applied PBL to high school further mathematics to find the effect of PBL on secondary students' achievement. They found out that students who used the PBL approach performed significantly better than those who were not taught using this approach.

Another study on PBL was done by Dahlgren (2000) on students undertaking a master's degree in computer studies. In this study the students were provided with content specific and detailed objectives. It was difficult for the learners to understand because the PBL was presented like a checklist. Dahlgren (2000) further contends that these reasearches were important to students because they demonstrated the importance of evaluating various ways of introducing a PBL approach in a classrrom invironment. Dahlgren (2000) then concluded that the relationship between the format of objectives and how students deal with them in the process of learning could also denote fragmented educational cultures within the three programs in computer studies, and may also lead to different interpretations of PBL in a learning environment. Therefore, the evaluation of PBL implementation is such a complex phenomenon because the objectives may be focused on process or content, whereas the outcomes may be relative to the pre-set objectives (Dahlgren, 2000).

The training of the Federal Bureau of Investigation (FBI) in American higher education, had been lecture-heavy and instuctor-centred during the impartation of knowledge to trainees (Barbian, 2002). For this reason, the FBI planned to shift the instruction approach to PBL in oder for the agents to have a better preparation of their daily job challenges and demands. Barbian (2002) argues that this change proved to be very effective in meeting this goal.

The PBL approach appears to have the potential to make the teaching of composition functions and inverses of functions more interesting, meaningful and to allow students to develop new strategies to solve other related real life problems. This learning strategy stimulates active learning among students which could stimulate the development of skills in “doing” mathematics rather than “knowing” mathematics, because it encourages students to be motivated and makes them autonomous thinkers who can openly communicate and collaborate with others (Savery and Duffy, 2005). Based on these findings, it can be stated that PBL is likely to improve foundation students’ understanding of compositions and inverses of functions.

After reviewing the literature, Colliver (2000) found no convincing evidence, at least in medicine, that PBL improves academic performance. Whilst PBL was a promising approach to educational innovation, implementation deficiencies could occur resulting in a lack of direction for both students and the facilitator and hence the rationale to repeatedly test the efficacy of the approach and map more accurately the conditions pertaining to any apparent success . Academic institutions worldwide are advocating for learner-centred approaches to teaching, and PBL is one of those teaching approaches which have the potential to change students’ performance.

2.7.3 PBL in mathematics learning and its implications in foundation education

Fatade, Mogari, and Arigbabu (2013) assert that the conventional teaching approaches which have been used for decades in higher education have been found to be limiting students in becoming proper problem solvers and self-directed learners. They further state that the early implementation of PBL in the further mathematics has been found to be one of the innovative approaches used in higher education. Bogdan, (2000) argues modern day educationists to emphasise more on teaching methods like PBL that allow students to be critical thinkers and real-life problem solvers. Through the implementation of the PBL approach, teachers have realised that students can engage in the professional environment not only to answer questions with regard to the problem or to reach at the correct solutions, but also to make research in ways that discover new explanations to unanswered questions (Gibbings, 2008). Thus PBL can be used as a tool that promotes the development of critical thinking skills among FP students.

2.8. DIFFERENT MODELS OF PBL APPLICABLE TO FOUNDATION PROGRAMMES

2.8.1. The foundation model approach

The PBL foundation model is almost similar to the funnel model described below (see 2.8.3). The introduction of lectures at the initial stages is done as a way of changing the way learners think before the introduction of a more innovative teaching approach like PBL (Nguyen, 2009). In the foundation approach, the knowledge gained by the students during the lecture method will form the firm academic base upon which more concrete work and difficult problems taught through PBL can be built. In the foundation approach, the teacher still takes the same role of being a facilitator of the learning process who keeps directing the learners throughout the entire learning process.

2.8.2. Barrows' Seven Jump tutorial model

The Barrows Seven Jump model is a PBL model which provides a stimulus for self-directed learning of the students following well-defined steps. In a seven jump approach, problems promote students' critical thinking and speculation about various ways of working on a problem and constructing new knowledge in the process (Zieber, 2006). As stated by Savin-Baden and Major (2004) the seven jump PBL model has the following seven steps: (i) *Clarifying* terms: Students are expected to make clarifications of any unfamiliar terms after being presented with a problem. This is for them to agree on the various words and terms that are used in the problem. (ii) *Defining the problem*: In this phase the group should discuss the definition of the problem and reach an agreement on tricky events which may need clarifications and elaborations. (iii) *Brainstorming:* Aspects on prior knowledge are collected at this level. Group individuals may discuss their ideas freely and openly. During the brainstorming process, the group collects as much information as possible, both relevant and irrelevant. (iv)*Structuring and hypothesis:* In this step, the group members will rearrange steps (ii) and (iii) into explanations for tentative solutions. (v) *Learning objectives:* At this level the students will formulate learning objectives and agree on them with the facilitator, ensuring that they are focused, achievable and measurable; comprehensive and appropriate. (vi)*Searching for information*: This is a stage where students employ their self-independent learning. They go home and search for information that can help them to address the problem. This phase is supposed to provide answers to the questions evoked in the problem-analysis phase. They also agree on how they will present their findings in the next lesson. (vii) *Synthesis:* The group shares the findings of the private study. The tutor checks learning and makes an assessment of the group. The acquired new knowledge is then tested with regard to the original problem investigated (p.3). In this model the facilitator guides the tutorial group successfully and may ask questions to draw attention to inconsistencies, widen the discussing and stimulate the integration of knowledge. At the end of the seven stages, it is the responsibility of the minute secretary to make a write up of the summary of the investigation and conclusions of the group.

The students will then provide feedback on their findings and evaluate the course as well as the quality and quantities of the group findings and procedures (Schmidt & Norman, 2003).

2.8.3. The funnel approach

In this approach students are first taught using other conventional methods like the lecture at lower year for tertiary students. A more directive and collaborative teaching approach is then applied later to help students to understand a particular concept at a higher level (Savin-Baden and Major, 2004).The funnel model incorporates PBL and other conventional methods for them to complement each other and improve student understanding (Dejan, Kirebi, & Molatu; 2013).

2.8.4 Single module model

This model is used at McMaster University. In this model, students gather in groups and are showered with problems to work on. The facilitator may be there or may not be there to support and redirect them when they get lost (Savin-Baden and Major, 2004). Very few lectures are provided to the students, but it has the advantage that the students are privileged in that they can work on problems on their own.

2.8.5 The two strand model

With the two strand model, learners make use of both the PBL model and other methods being employed concurrently (Nguyen, 2009).This model is used if learners have to share modules across disciplines. Therefore modules in each strand according to Savin-Baden and Major (2004) are planned with linking topics in order that information attained from the mixed approach becomes supportive for students in the PBL process.

2.8.6 The Shoestring Problem –Based learning model.

This model is applied right across all the academic year levels of a university student's entire programme duration. It has a mixture of PBL and lecture method applied together during the course of study of a particular student (Savin-Baden and Major, 2004). With this model, the designed problems are based on a specific subject being taught and during the application of PBL, the teacher takes the role of a facilitator to give support and raise the academic morale among the students and reduce frustrations. Lectures are scheduled so that learners can be kept guided throughout the course of their study. Savin-Baden and Major (2004), state that learners in this model may not understand the rationale behind the dual implementation of PBL and the lecture method if they are not well supported. Table 2.4 below shows a clarification of how the PBL shoestring model may be implemented across different year levels.

Table 2.4: Example of PBL on a shoestring (Adapted from Savin-Baden & Mayor, 2004, p. 38)

Year 1	**PBL**	Lecture	Lecture	**PBL**	Lecture
Year 2	Lecture	**PBL**	Lecture	**PBL**	Lecture
Year 3	Lecture	Lecture	Lecture	**PBL**	**PBL**

2.8.7 The integrated, complexity and patchwork models of PBL.

Savin-Baden and Major (2004) propose the integrated, complexity and patchwork models as other approaches to PBL. In these models, the students work on sequential problems across boundaries. However, the complexity model is the most complicated of the three because it is used to design programmes “that transcend subjects, disciplines and university curriculum impositions, and embrace knowledge, self-actions, and curriculum organising principles.

2.8.8 Other PBL Models

There are other models of PBL implementation Duch (2001), which can be employed to undergraduate or foundation students. These include: “the medical school model”, “the floating facilitator model”, “peer tutor model”, “large class model”. These models are seen to have similar features to those used in medical schools except for the floating facilitator model which is used for multiple groups of learners in very large classes. In this model, the tutor is considered as floating because he/she moves around to facilitate group work (Nguyen, 2009). When groups are many and classes are large, a “roving facilitator” may be introduced who may be a senior student, not necessarily a subject expert who will be facilitating group proceedings.

Following the discussion on the various PBL models above, the shoestring model looks more applicable to a bridging course for undergraduate mathematics. This is because the model can be used for various units at different grades or academic year levels (Lambros, 2004). It is also suitable for larger classes, and for that reason the shoestring model allows the use of the lectures which are suitable teaching approaches for large class sizes. The shoestring model is also suitable for learners who are new to the PBL approach since it still incorporates the lecture methods which are the conventional approach students are conversant with. This model gives students an opportunity to slowly adapt to the PBL approach. Nguyen (2009) affirms that the shoestring model is suitable where there is a single tutor facilitating the group activities and the tutor can also incorporate the lecture method for maximum benefit of the students. With the PBL shoestring model, learning problems can be designed based on specific disciplines and lectures can be scheduled as ways to facilitate learning (Saarinen-Rahiika, Binkley, & Hayes, 2008). The PBL shoestring model is relevant to FP students because it uses the lecture method to lay the academic base then the PBL approach will follow when the proper base has been laid. In the present study the researcher combined the Burrows seven jump model and a modified shoestring PBL model. This hybrid model was found to be suitable for the one year SFP at Oshakati campus because both the researcher and the FP students were new in the application of the PBL approach and the researcher still needed the lecture method to supplement his implementation of the PBL approach. Apart from that, the use of the hybrid model was good for their adjustment.

2.8.9. Hybrid Model adopted in this study

Science Foundation Programme is a one year course as highlighted in chapter one. The shoe string model for PBL is supposed to be applied years to students as stipulated by Savin-Baden and Major (2004). A modified shoestring model where PBL sessions will be applied in weeks will suite the bridging course for undergraduate mathematics.Year1 will be mean by Week 1, and Year 2 to mean Week 2 etc. Table 2.5 presents the modified shoestring model adopted in the study

Table 2. 5: Modified Shoestring model used in the study

	Session 1	Session 2
Week 1	**PBL**	Lecture
Week 2	Lecture	**PBL**
Week 3	**PBL**	**(Lecture) PBL**

In each of the PBL sessions Barrows Seven Jump approach was used to solve problems and giving feedback. This hybrid model was suitable the SFP because it's a one year course where mathematics is taught by one lecturer and the original shoestring model would not be inapplicable since it requires more than three years with different facilitators in each year level. The facilitator and the students were both new to the implementation of PBL; therefore the incorporation of projects, tutorials and group work made the PBL sessions more meaningful especially where there was need for clarity. Apart from that, in a FP it would be good not to employ PBL in every topic but to some topic in order to have time to cover the foundation mathematics syllabus. Therefore the use of the hybrid model necessitated that.

2.9 CRITICISMS OF THE PROBLEM-BASED LEARNING APPROACH.

While PBL has been found to have several benefits to students, it also has its own shortcomings. Since PBL focuses on specific problems it may compromise the breadth of the content coverage (Kain, 2003). Academic achievement scores favour traditional teaching methods when standardised tests are used but favour neither PBL nor traditional methods when non-standardised assessment forms are employed (Vernon and Blake, 2003). Vemon and Blake further state that traditional methods are valued better when it comes to coverage of science and mathematics content areas and in evaluating students' knowledge of content.

Kain (2003) contends that students sometimes find it difficult to build up over layers of knowledge from simple to sophisticated concepts as the case is in traditional instruction but in PBL they only gather what is relevant from knowledge to solve traditional problems. However, PBL tends to reduce initial levels of gaining relevant knowledge; it improves long-term retention (Kirschner et al., 2006). PBL has shown that it increases students' ability to solve real-life problems better than traditional approaches. However, modern instructors don't seem to have soundly supported this type of teaching approach because it requires a lot of time for preparation and planning, rendering it an inefficient approach in an educational set up (Delafuente et al., 2004). Therefore in an FP it would not be good to employ PBL in every topic but to some topics in order to have ample time to cover the foundation mathematics syllabus.

One of the expectations of PBL according to Albanese and Mitchell (2004) in terms of students' role is that, students learn what they want to learn and the teacher is just there as a learning facilitator. With regard to this aspect, students may not know what is relevant for them and what may not be relevant to them such that in the end they may waste time emphasising on what might not be of improtance, which may not even be the thrust for examination (Kirschner et al., 2006). As already noted earlier (see section 2.2.2) in traditional approaches, the teacher is perceived as the main source of knowledge and the information disseminator but in a PBL classroom its vice versa. As such, this orientation, that the teacher is the expert in information delivery in traditional settings, many students appear to have lost their ability to "simply wonder about something" since in a PBL set up they are the ones to set the pace. Therefore the FP lecturers should employ their expertise to guide and direct students in their problem solving skills.

Since there is a reversed role of the teacher and the student in a PBL approach, the instructor has to alter his/her traditional teaching methods, lectures, discussions and asking students to memorise for the tests (rote learning). In a PBL approach, the teacher is more of a facilitator of the learning process than the disseminator of information. As a result, the instructor acts and focuses his/her attention on questioning student logic and beliefs, providing hints to correct erroneous student reasoning, providing directions and keeping students on the task (Woods, 1994). This responsibility is foreign to some teachers and they may have trouble in breaking with their past (Delafuente et al., 2004). However, one common criticism of the PBL approach is that students cannot know what really might be important for them to learn, especially where they have very little or no prior knowledge and as such, teachers as facilitators must be careful to assess and account for the prior learning that students bring to the classroom.

Another shortcoming is that the application of the PBL approach requires a lot of time in setting the group activities and planning and preparing the relevant problems that should be used as starting points by the teacher, therefore this approach compromises the completion of the syllabus when compared to the conventional lecture method (Dolmans et al., 2005).Therefore careful planning must be done by the FP lecturer so that PBL does not compromise the completion of the foundation programme syllabus. Schmidt (2003) has noted that the implementation of the PBL approach may be difficult for the teacher to “relinquish control” and become a facilitator, encouraging the students to ask the right questions rather than handing them solutions. The lecturer can overcome this shortcoming by attending PBL workshops that will help lecturers and teachers with PBL implementation strategies.

The generation of tasks is a critical requirement for PBL. Problems should encompass both the large goal and specific objectives which students must find on their way to reaching the goal’s solution (Schmidt and Norman, 2003). A study by Dolman et al. (2002) on student objectives and PBL application showed that students did not stay on track and omitted most of the important objectives. Conoldi (1998) also notes that if students divert from their anticipated directions during their solution generation, they may completely miss the main content if the facilitator does not redirect them well.

Another criticism of the PBL approach is based on the assessment criteria used. Schmidt and Norman (2003) advise that student knowledge and achievement may be measured using alternative forms which may include written examinations, practical examinations, concept maps, peer assessment, self-assessment, tutor assessment, oral examination and written reports. Assessment practices often mismatch innovative approaches by favouring traditional pen and paper exams. At Stellenbosch University 60% of the continuous assessment tries to assess the shortcomings and gives the lecturer the latitude to implement peer assessment which is crucial to ensuring effective group involvement in PBL (Malan, Ndlovu, & Engelbrecht, 2014).

Despite the many benefits of PBL pedagogies, PBL is still not widely used in K-16 classrooms in American classrooms. One serious challenge faced by even expert practitioners of PBL is the intense demands this pedagogy places on teaching and learning. It takes time to develop good problems and more time to facilitate the sessions (Marsteller, 2008). Chapman (2011) argues that designing a problem involves choosing an authentic real-world problem that the students can relate to. A problem has to require some final product(s). A good problem has to be designed to support multiple hypotheses, present itself without too much “given” information, be open-ended, and “messy” (Grady, 2010). Therefore FP students need to be taught to design and analyse a problem, identify relevant facts and generate facts and hypothesis, identify necessary information for solving the PBL problems and make reasonable judgements about the relevance of the problems. In addition, students work on problems in small groups, requiring the lecturer/teacher to devote significant amounts of time guiding students through the PBL process. In larger classes, student ratios do not permit this level of individualized guidance.

In conclusion, one can state that the teaching approach that is adopted by a teacher will determine the students' level of comprehension and retention of the taught material. Modern employers have appreciated the positive attributes of improved communication, teamwork; respect and collaboration that PBL experienced students have developed. These multi-skills provide for a better future skills preparation in the ever-changing information explosion. The PBL curriculum includes building these attributes through knowledge building, written and interpersonal interactions and through the experience the problem solving process.

2.10. SOME CONSIDERATIONS IN THE IMPLEMENTATION OF PBL

2.10.1. Students' roles and responsibilities in PBL

As stated earlier on, learning in a PBL approach is done in small groups of 5 to 8 people. The students work in these groups to analyse, discuss and engage in self-directed learning (Dolmans et al., 2005; Barrows , 2002). In these groups, the students will select the group spokesperson, the leader, the time keeper, and the group secretary. The group leader moderates the brainstorming and moderates the groups' proceedings to keep the group on track and is actively involved in group discussion (Belland, French, & Ertner, 2009). A new group leader may be selected on each new session so that all members of the group have an opportunity to be leaders at the end of the PBL implementation. The time keeper who is also a nominee of the group is responsible for monitoring time and making sure that the progress through the steps and the case-content is balanced leaving sufficient time for a thorough discussion on one hand but being fast enough to complete the task. The secretary for the group members is responsible for writing down the team's discussions and documents the learning goals and objectives assigned by the group participants. The secretary files and keeps the record of all the group proceedings.

2.10.2. Teachers' roles and responsibilities in problem-based learning

Barrows (1996) asserts that the tutor should have expertise in organising and facilitating group activities rather than subject area (content expertise). These tutorial sessions are viewed by Lohman (2004) as professional strategy meetings. In PBL, the tutor facilitates or activates the group to ensure that students' progress satisfactorily through the problem solving process. Roh (2003) recommends that the tutor does this by questioning, probing, suggesting, encouraging critical reflection, as well as challenging in a helpful and constructive way but only where necessary. Most of the modern tutors are challenged by "where necessary" as they cannot make a distinction as to when it's necessary to intervene in a PBL facilitation (deciding when and how) part of the intervention (Barrows, 1996). At McMaster University, Woods (1994) emphasised that the tutor must be skilful in facilitation, active learning, listening, motivating learners, critical reflection and must not dominate the session with content specific questions and answers that convert it into a tutor-led seminar. Therefore the tutor is the custodian of the group proceedings and guides them for discovery, rather than an information dispensing model of perfection or an over-enthusiastic educational leader (Barret, 2004). The tutor and the student should act like colleagues and not that of a minor and a superior (Savin-Baden and Major, 2004; Savery, 2006; Saarinen-Rahiika et al., 2008). Such a working relationship will enable the students to freely express their ideas and communicate how they feel about a particular idea unlike where there is a minor-superior relationship where superior is known to be the main source of information.

2.10.3 Problem-based learning curriculum design factors.

Barlett (2005) emphasises that when starting a PBL initiative, it is critical to be aware of the success factors affecting PBL implementation. In designing a PBL curriculum, the following four important considerations identified by Barlett (2005) need to be met: content, teaching and learning strategies, assessment processes and the evaluation process. The process of organising these elements into logical PBL patterns is called curriculum design (Bartlett, 2005). The design of a PBL curriculum takes either a prescriptive model (a model which indicates what curriculum designers should do), or a descriptive model (a model that purports to describe what curriculum designers actually do). Pawson et al. (2006) attest to the fact that the incorporation of a PBL curriculum in mathematics represents a difficult task. Their concern was that:

> "In problem based learning…..the knowledge must often be acquired in advance of participation in the problem solving process" (p.104).

The application of PBL in the foundation programme does not necessarily presume the complete abolishment of other traditional instructional techniques like lecturing for the purpose of enabling students to acquire the necessary prior knowledge, PBL has great implications to curriculum changes which need to be put into consideration prior to implementation.

Azer (2005) identifies the following factors critical to the implementation of the PBL: (i) Preparing students fully for change, (ii) preparing a new curriculum committee and working group, (iii) designing the new PBL curriculum and defining educational outcomes, (iv) seeking advice from expert PBL personnel, (v) planning, organising and managing. (vi) training PBL facilitators and defining the objectives of a facilitator, (vii) introducing the students to the PBL programme, (ix) using learning to support the delivery of the PBL programme. (x) changing the assessment to suit the PBL curriculum, (xi) encouraging feedback from students and teaching staff, (xii) managing learning resources and facilities that support self-directed learning, (xiii) continuing evaluations and making changes (p.809-812).The implementation of the PBL of the current study was in line with Azers' (2005) identified eight identified factors for successful implementation of PBL.

Barret (2005, p. 4) contends that the design of a PBL curriculum should seek to answer the following four questions according to:(i) What educational purposes should the institution seek to attain (the product) (ii) What educational experiences are likely to attain these purposes (iii) How can these educational experiences be organised effectively (iv) How can we determine whether these purposes are being attained? Therefore, according to Barret the most important aspect is the educational purpose of the institution. As highlighted earlier (see 1.1.4) the role of the FP is to equip disadvantaged students with scientific skills that will allow them to be admitted at tertiary institutions. Therefore in line with this national goal of increasing university access, the implementation of the PBL approach at FP will help to meet this goal.

2.10.4 The nature of assessments in a PBL approach

Pawson et al. (2006, p. 106) caution that PBL is an instructional approach which was meant to assist those individuals “who spend their professional lives toying with problem scenarios”. There are two forms of assessment, namely authentic and traditional that can be used in the classroom. Savin-Baden and Major (2004, p.119) contends that traditional assessment “is grounded in the belief that knowledge is universal and that students can take knowledge as it is disseminated”. An example of traditional forms of assessment is learning by rote learning and then simply reproducing the learned knowledge through “multiple choice examinations, true or false questionnaires or filling in blank spaces in quizzes and puzzles and test assessments” (Savin-Baden and Major, 2004, p.119). By contrast, PBL should focus on performance-based assessment rather than standard quizzes and test assessments. Lambros (2004) asserts that because PBL involves flexibility and creativity, assessment measures should possess similar qualities. In this study the use of the research project on the real life application of compositions and inverses of functions was in line with the goal of allowing FP students to demonstrate their understanding of the topic taught using the PBL approach.

With regard to authentic assessment, Savin-Baden and Major (2004, p. 119) challenged the idea that knowledge is universal, stating that “knowledge has multiple meanings and as such, cannot be measured objectively”. This view is consistent with the flexible, process-oriented and student-centred nature of PBL. Therefore assessment measures in PBL are based on the progress of each individual student (Pawson et al., 2006). Chapman (2013) contends that if students have done some a group e.g each student writing a chapter or a section of a resaerch project in line with PBL each member of the group will get an individual mark. How the chapter or sections link up to form a coherent whole can be a mark assigned to the group. Lambros (2004) recommends that assessment in PBL should include the whole learning process and not just the final assessment of knowledge acquisition as in other approaches. He further argues that assessments in PBL should occur as students’ progress through activities and end with a "culminating activity or demonstration of achievement" (p. 57). Lambros (2004) identified two forms of assessments that are relevant to the PBL approach: self-assessment and peer assessment. These two forms of assessments in PBL will allow the student to:

> "Use the same form and criteria that [the teacher will use and] have the chance to see how well their assessments align with [the teacher]" Lambros (p. 67).

Lambros (2004) further argues that self-assessment reflects the subjective nature of PBL because students are not only responsible for their own learning but are also measuring their own progress since PBL self-assessments and peer assessments have real-life implications. Savin-Baden and Major (2004) summarise other forms of assessments that can be used in the PBL approach: group or individual presentations, tripartite assessment, case-based individual essays, case based care plans, portfolios, viva voce examinations, reflective (online) journals, reports, patchwork tests, examinations and electronic assessments. This study adopted individual group presentations, portfolios and patchwork tests which were administered to two groups of students the PBL and the comparison group.

2.11. THE LINK BETWEEN PBL AND REALISTIC MATHEMATICS EDUCATION

Realistic Mathematics Education (RME) is a teaching and learning theory in mathematics education that was first introduced and fully developed by the Freudenthal Institute in Netherlands and has long since been adopted by a large number of countries worldwide such as Malaysia, Spain, Brazil, Japan, Denmark, Portugal and RSA (De Lange, 2006). Freudenthal (1991) asserts that mathematics is a human activity that must be connected to reality by whoever wishes to pursue it. Freudenthal (1991) further posits that mathematics must be kept close to the children and it must be relevant to everyday life situations. The word "realistic", does not only refer to the fact that mathematics must be connected to the real world, but also refers to problem situations which are real in the minds of students including fantasy that may be of interest to the learners (De Lange, 2006). This means that for these problems to be presented to students, the context can be a real-world context though it's not always a necessary condition. However the application of modelling can also be used in problem situations (Freudenthal, 1991).

Freudenthal's idea of mathematics as a human activity simply meant that mathematics education must be organised as a process of guided reinvention, where students can experience it as a process similar to that by which mathematics was historically invented (De Lange, 2006). The heart of PBL is centred on the fact that students should be effective real-life problem-solvers (Barrows, 1996). It can be seen that the goal of Freudenthal's RME and Barrows' PBL is the same, that of making students autonomous and independent problem solvers in their lives.

In RME, Treffers (1987;1991) identified two types of mathematization which can be formulated in an educational context and these are horizontal and vertical mathematization. Treffers further argues that in horizontal mathematization, students come up with mathematical tools which can help them to organise and solve a problem located in a real-life situation. In horizontal mathematization, students identify or describe specific mathematics in general contexts; schematizing, formulating and visualising a problem in different ways, discovering mathematical relations and regularities, transferring a real-world problem into a mathematical problem, transferring a mathematical problem into a known mathematical problem (Zulkardi, 1991). On the other hand, vertical mathematization is the process of reorganisation within the mathematical system itself. Zulkardi (1991) further states that in vertical mathematization, students represent relations in a formula, formulate mathematical models and generalise, refine and make adjustments to models and use different models. Freudenthal (1991) further explains that horizontal mathematization involves going from the world of life into the world of symbols, while vertical mathematization means moving within the world of symbols. He adds that the distinction between the two types of mathematics is not always clear cut.

Freudenthal (1991) classifies mathematics education into four approaches which are: mechanistic or drill and traditional approach - which is based on drill and practice of the learner; the empiristic approach – which treats the word as a reality in which students are provided with materials from the living world where they perform their horizontal mathematization; structuralist approach - where the students perform horizontal mathematization through the use of charts, puzzles and quizzes; realistic approach - this is an approach where students deal with real world situations or problems which they explore through horizontal mathematization.

In all, it is justifiable to conclude that the role that PBL and RME play is substantially similar as both stress problem solving and modelling real (or realistic) situations. Both educational approaches also aim to produce autonomous learners which are a desirable attribute for student success in higher education.

2.12. SUMMARY

The literature review in this chapter started off by defining PBL and describing its characteristics. The literature was also used to explore the philosophical principles underpinning the PBL approach and FP. Emphasis was placed on how PBL can enhance the metacognitive aspects of student understanding of mathematics in a bridging course for undergraduate mathematics. It then explored aspects on teaching and learning theories. The literature was used to precisely locate the PBL approach with the learning theories. Apart from that, the literature review was used to explore how study methods link with a particular teaching method. It then explored the various models of PBL that are often used and can be applied to a SFP

The literature was used to explore the various definitions of the PBL approach, followed by an exploration of the origins of PBL. Apart from that, this chapter used literature to review the potential role of PBL in higher education in general and mathematics teaching in SFP in particular. Some emphasis was placed on the implementation and essential features of PBL and its aim in a FP. The researcher also used literature to explore the evaluations, curriculum design, assessment and implementation of PBL in an educational set up and then looked at some studies where PBL was successfully implemented and where PBL implementation was not a success. The aim of PBL and its criticisms were also explored using the review of literature. I addition to that, the researcher also used the literature explore the various roles for students and the teacher in a PBL set up.

Furthermore, literature was also used to explore the various models of PBL and their implementation which are in existence. The philosophical principles underpinning the PBL approach were also explored in the literature review. The review of literature was also used to clarify and compare realistic mathematics education by Freudenthal and PBL by Barrow. This research study has two chapters for literature review. The next Chapter reviews the literature on the historical development of the function concept, the teaching and learning of functions in general and compositions and inverses in particular.

CHAPTER THREE:

THE FUNCTION CONCEPT AND THE TEACHING AND LEARNING OF COMPOSITION AND INVERSE FUNCTIONS

3.1. INTRODUCTION

This Chapter will review literature on the evolution of functions as a concept for mathematics and will then explore the role that is played by functions in the teaching and learning of mathematics. Apart from that, it will give the different frameworks which are used by students to understand functions. The literature will also be used to explore previously undertaken studies on functions. This chapter will use literature to explore the Hypothetical Learning Trajectory (HLT) together with its underlying features. Finally this chapter will present the conceptual framework used in the study and how it was adopted in the study.

3.2 EVOLUTION OF THE FUNCTION CONCEPT

Kleiner (2003) indicates that the evolution of the function concept dates back 4000 years ago. Kleiner further claims that this concept is intimately connected to calculus. On the other hand Schaaf (1930) suggests that the concept "function" marks a distinction between "classical" and "modern" mathematics. He goes a step further and states that:

> "The keynote of the Western culture is the function concept, a notion not even remotely hinted at by earlier mathematical culture. The function concept is anything but an extension or elaboration of the previous number concept – it is rather a complete emancipation from such notions" (p. 500).

Kleiner (2003) explains that the evolution of the function concept can be seen as a tug of war between two elements, two mental images: the geometric (expressed in the form of a curve) and the algebraic (expressed as a formula-first finite and later allowing infinitely many terms, the so-called "analytic expression"). A logical definition of a function according to Kleiner (2003) is a correspondence with a mental image of an input-output machine. This definition suggests that a function can be viewed from the perspective of an "action and a reaction" the input variable can be viewed as the action and the output can be perceived as the reaction that comes as a result of the action. In the 17th century Isaac Newton (1642-1727) whose principal objects of study were geometrical curves, and the variables associated with a given curve, were geometrical quantities: co-ordinates, tangents, lengths of a chord etc. (Kleiner, 2003). The exception that appears in Newton's development of calculus is the fact that all variables were (essentially) regarded as functions of time.

Later on, Leibnitz (1646-1716) introduced the word "function" into mathematics. For him this word always stood for a geometric object associated with a curve, e.g. "the tangent as a function of a curve" (Kleiner, 2003). As time unfolded, Johann Bernoulli (1667-1748) placed more emphasis on the use of formulas and equations relating to 'functions' of a curve and made the definition of a function more algebraic. As stated by Kleiner (2003). Bernoulli defined a function as follows:

> "A function of a variable is a quantity composed in any manner whatsoever of this variable and of constraints (p.1)".

Bernoulli did not clarify what he meant by "composed in any manner whatsoever". The word whatsoever used by Bernoulli to define a function might have meant that the function could be exponential, quadratic, trigonometric, logarithmic, etc. as long as there is some relationship that can be traced between the existing variables or constraints.

Later on Leonard Euler (1707-1783) defined a function as:

> "An analytical expression composed, in any manner, of the same quantity and of numbers or of constant quantities" (Grabiner, 1981, p. 51).

In his definition of a function, Euler included the following functions: polynomials, power series, trigonometric and logarithmic functions and also defined functions of several variables and classified functions as algebraic and transcendental (Lucus, 2003). According to this classification, transcendental functions were logarithmic, trigonometric, exponential and some integrals. Though Euler did not make specifications of what he meant by "analytical expressions", basing on examples given above, he would have regarded

$$f(x) = \begin{cases} -x & (x < 0) \\ x & (x > 0) \end{cases}$$

as defining two functions, not one. However, he later gave yet another definition of a function which still omitted the word "analytical expression". It states that:

> If two quantities depend on each other in such a way that if the latter is changed, the former undergoes changes themselves then the former quantities are called "functions" of the later quantities" (Kleiner, 2003) (p.2).

The definitions of a function which were given by Leibnitz and Euler have some similarities in them in that a function represented some form of a relationship between two or more variables which are related to each other. These definitions further suggest that given either the output/reaction one can trace back the input/action.

In 1810, Lacroix published a book entitled *Traite du Calcul Differential et Integral* in which he defined a function as follows:

> "A function is any quantity whose value is dependent on one or more [other quantities] variables, whether one knows or doesn't know the operations by which one passes from this [other quantities] to the first (Gardener, 1982: p. 271).

The definition of a function given by Lacroix does not have much difference with the definitions of a function already given earlier on except that the use of the word dependent clarifies that in function the variables are relying on each other in some way. Therefore the latter definition of a function sounds more explicit and complete.

Lucas (1996) states that in the 19^{th} century Lagrange was convinced by Euler's discovery about a function and gave his own definition of a function. He defined a function as any "*expression de calculi*" (p.17) into variable entered in any way. In 1813 Lagrange published a book entitled *Theorie des Fonctions Analytic* in which he gave the definition of calculus as the "theory of analytic functions" and further defined a function of one or more variables as "any analytical expression useful for calculation in which variables enter in any manner whatsoever" (Kline, 2005, p. 406). Lagrange's definition of a function also continued to have the word whatsoever which may suggest that the way variables may appear in a function could be quadratics, cubes, polynomials, etc.

Cauchy was a mathematician who did rigorous mathematical work in calculus. When he looked at the definitions of a function which had already been presented by Lagrange and Euler, he concluded that they did not seem to include the functions which represented differential equations from physics and mechanics, and did not comply with the definition that functions were considered as analytic expressions (Lucas, 1996). The definition by Lagrange for example was not satisfactory enough to incorporate all existing types of functions from Cauchy's perspective. Cauchy then defined a function in *Cauchy's' Cours d'Analyse* (1821) as follows:

> "The existence of a relationship between two variables, is in such a way that if one of them is given, one can derive the other variables based on what has been given, usually one of these variables as all being expressed in terms of the one, which is called the independent variable, and the other quantity is expressed in terms of the independent variable are what one calls functions of this variable" (Gardiner, 1982: p. 263).

The definition of a function which was given by Cauchy does not differ with the modern definition of a function which is defined in terms of dependent and independent variables. This definition excludes the word "whatsoever" which was used by earlier mathematicians, and is therefore more refined and more specific to what we know a function to be today.

While studying vibrating strings, Joseph Fourier (1768-1830) was led to consider a more general definition of a function. His definition stated that:

> "The function *f(x)* of *x* represents a succession of values or ordinates each of which is arbitrary. An infinite number of values being given to the abscissa *x*, there are equal numbers of ordinates *f(x)……*We do not suppose these ordinates to be subject to a common law; but they succeed each other in any common law whatsoever, and each of them is as if it were a single quantity" (Kleiner, 2003, p. 2).

The definition of a function by Joseph Fourier is more abstract and is defining a function as a set of ordered pairs represented on the Cartesian plane. All the previously given definitions did not include the abscissa and ordinate as was done by Fourier.

Kleiner (2003) asserts that there was no rigorous attempt to prove Fourier's results on vibration strings even by the standard of that time. Peter Lejeune-Dirichlet (1805-1859) was one of the many people who worked to complete Fourier's proofs. Dirichlet's definition of a function as an arbitrary correspondence was not very different from the definitions of functions which were given before him. The only difference as stated by Kleiner (2003), was that Dirichlet's function defined as

$$f(x) = \begin{cases} 1 & : x \quad Rational \\ x & : x \text{ is irrational} \end{cases}$$

was neither given as an analytic expression nor expressed by a geometric curve.

Kleiner (2003) claims that functions did not emerge until the beginning of the 18th century, though their manifestation can be dated back to about 2000 BC. As stated by Kleiner, there are several reasons why functions did not emerge in earlier mathematics. Some of the reasons why functions did not emerge before the 18th century are as follows: lack of algebraic prerequisites – coming to terms with the continuum of real numbers, and the development of symbolic notation were some of the reasons why functions did not feature in ancient mathematics. However, between 1450 and 1650 there occurred a number of developments which were fundamental to the rise of the function concept (Kleiner, 2003). The following were some of the reasons: (a) extension of the concept of number to embrace real and (to some extent) complex numbers (b) the creation of symbolic algebra (c) the study of motion as a central problem of science (d) the wedding of algebra and geometry (Kleiner, 2003, p. 2).

The history of compositions and inverses of functions can be traced back to the time of Dirichlet (1805-1859). The two concepts are treated as related to each other. Composition functions are treated from a computational point of view, for example, using polynomials and rational functions, arrow diagrams, ordered pairs or radical functions. Inverse functions are defined as special functions that *undo* each other (Boogni, 2009).

3.3 THE ROLE OF FUNCTIONS IN MATHEMATICS TEACHING

The function concept is a very important component in the teaching and learning of mathematics from primary school, till a student finishes high school since it equips students with the proper skills to solve problems in calculus and other related concepts (Carlson and Oerthman, 2005). However, at the FP these concepts are taught in an abstract format preparing students for university basic mathematics, pre-calculus and calculus. Students who have a good understanding of these concepts stand a better chance of performing well in their mathematics at higher levels (Even & Bruckheimer, 2004). Kimani (2008) contends that functions are central to many branches of mathematics and if a student has a deep understanding of functions, he will be able to quickly make mathematical connections that are needed in understanding scientific and other related concepts. Cooney, Beckman, Lloyd, Wilson, and Zbiek (2011) also note that students with a deep understanding of functions are able to represent them in graphical notation, switch from one function to another, express functions verbally and represent them in the form of an equation. Students who demonstrate these representations will understand better relationships between or among variables, and increase their ability to solve problems in real life situations, which is a critical skill of the study of mathematics.

A strong understanding of the function concept will help students to understand the basics of calculus (often referred to as the mathematics of motion or change) which is a component of mathematics applied to science and engineering (Kleiner, 2003). Functions together with their compositions and inverses are important a Foundation programme because the FP students intend to pursue science and mathematics degrees in future where they will encounter pre-calculus and calculus. E.g.

Given that $\mathrm{f(x) = 2x - 6}$ *and that* $\mathrm{g(x) = e^{3x-6}}$ *find:*

i. $\frac{d}{dx}(fg(x))^{-1}$

ii $\int gf(x)dx$.

Student with strong understanding of functions and their compositions and inverses it will not be difficult them to hand the problem above. Therefore the teaching and learning of functions (compositions and inverses) will prepare FP students fully for university mathematics. Cooney et al. (2011) argue that students who connect functions to a formula develop a primitive understanding of functions as they connect composition functions with substitution only. For example, in the illustration to find the integral of $f(x)=2^x$, $g(x)=2x+3$, if asked to find $fg(x)$

$$\begin{aligned} fg(x) &= f(g(x) \\ &= f(2x+3) \\ &= 2^{2x+3} \end{aligned}$$

In the illustration above, the students should understand a function from different perspectives and be able to manipulate a complicated function and differentiate even an exponential function from first principles. A student with such an understanding of a function will be better able to handle complex engineering mathematics problems.

Lucas (1996) is of the view that in the 18th century functions were identified with their formulas, which were infinite (endless) and relatively "well behaved". With regard to the definition of a function, Grabiner (1982) suggests that:

> "The difficulty inherent in making this translation from specific examples of what we would now call "functions" to an adequately general function-concept was one of the main predicaments in the way of explaining precisely why, when and how methods of calculus could be trusted" (p.255).

This is evidence to the fact that without functions, calculus would be difficult to explain and define. Therefore it is legitimate to claim that the study of functions is an indispensable prerequisite to the successful study of calculus in mathematics. The inclusion of functions (compositions and inverses) in the Foundation Programme is meant to meet this goal.

In order to equip students with the proper skills and technical knowhow to understand functions well, there have been calls from the Namibian educational curriculum developers to include function related activities as early as grade four (see 1.1.5) (MoE; 2006; 2007; 2008). Apart from that, the Ministry of Education (MoE) for Namibia, together with the National Institute for Educational Development (NIED) expects Namibian High school students to be able to understand and perform operations on functions. For example, finding their sums, products, differences, quotients, performing transformations of functions, inversion of functions and formulating composition functions (MoE, 2008).

If students are taught composition and inverses of functions using the PBL approach and demonstrate flexibility – visualizing one of them when working with the other in a PBL approach, it can motivate them to want to learn more about functions or related concepts. That will ultimately help them to develop their problem solving abilities in real-life and future mathematics endeavours (Carlson, Smith, & Persson, 2003).

3.4 THEORETICAL FRAMEWORKS FOR STUDENTS' UNDERSTANDING OF COMPOSITIONS AND INVERSE FUNCTIONS IN A FOUNDATION PROGRAMME

Many researchers (Asiala and Carlson (2006); Cotril (1999); Dubinsky and Weller (2013); Confrey and Costa (2006); Tall and Vinner (1981)) have identified different frameworks to explain students' understanding of compositions and inverses of functions. The most commonly used conceptual frameworks for the teaching and understanding of functions appear to be: (a) the flexibility framework, (b) the structural-operational framework, and (c) reflective abstraction or the Actions, Processes, Objects and Schemas (APOS) framework. However, there are other frameworks which are also used though not commonly and these include (d) property-oriented function view (e) function model framework and (f) prototypes, multiple representation transformation framework.

3.4.1 The Flexibility Framework of understanding functions

The flexibility framework is one of those frameworks which have been utilized by researchers in order to investigate students' understanding of functions. In the flexibility framework, functions can be viewed from two major perspectives: the function view which could be object or process, and the different representations which could be algebraic (symbolic), graphical, verbal or tabular (Kimani, 2008). Foundation teaching of compositions and inverses of functions should foster students' understanding of functions from different views. Students' understanding of functions varies depending on how they have been taught and depending on the dimension the teacher would have pursued (Moschkovich, Schoenfeld, & Arcavi, 2003). Foundation mathematics teachers should adopt different teaching approaches which will allow students to understand compositions and inverse of function from different dimensions.

Kimani (2008) stresses that flexible utilization of the different views of functions should be used in the classroom so that students can understand functions symbolically, diagrammatically or the tabular form.

For example, the function represented symbolically/ algebraically as $f(x) = x + 6$ can also be represented graphically as in Figures 3.1 below.

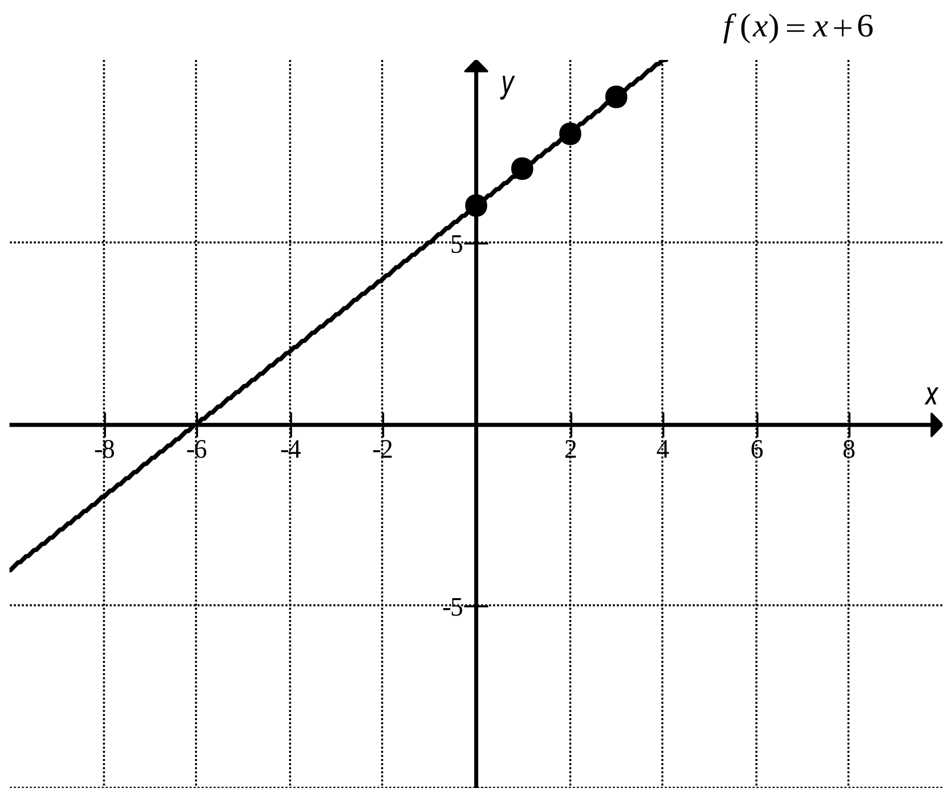


Figure 3.1: Graphical (Object) representation of the function f(x)=x+6

The same function above can be represented by a mapping diagram as in Figure 3.2 below.

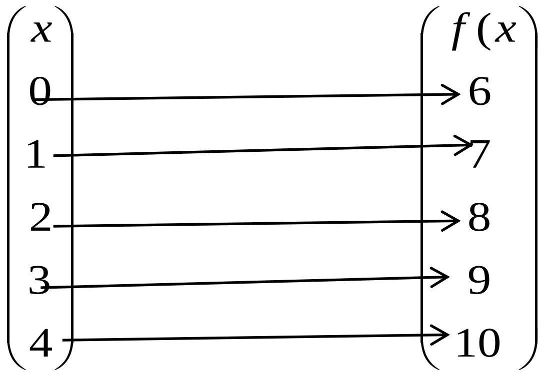


Figure 3.2: Mapping for the function f(x) =x+6 (definition of function as a Process

When students have adopted all these perceptions, they are exhibiting versatility and adaptability in their understanding of functions. Kimani (2008) points out that in the adaptability aspect, students choose the most efficient perspective of a function (object or process) and combine it with the most efficient representation to solve problems related to functions, whereas the aspect of versatility in understanding functions is exhibited when the student possesses different perspectives of a function (object of process) as well as interpreting and representing functions in a variety of ways as in the diagrams above. Some researchers use versatility and adaptability in an analogous manner, explaining students' versatility as the ability to view a function as a process in one context and in another context as an object resulting from the process, and adaptability as the ability to apply the proper perspective to the task at hand (Moschkovich et al., 2003). Even (2005) argues that connections between different modes of representation influence mathematical learning and strengthen students' ability in using mathematical concepts of functions. Therefore SFP teaching should be focused towards developing FP students understanding of a function from different perspectives for them to have a strong mathematical base for university mathematics.

Students who are able to profitably employ the object (representation) framework can achieve a deep and coherent understanding of functions than when they employ only the algebraic approach which gives them a mere local image of the concept of functions (e.g. $f(x)=x+6$) gives a local image which is an algebraic representation of a function. For example, as in the above illustration, some students cannot make sense of the graph of the function $f(x) =x+6$ and yet they claim that they understand functions. If a student can represent $f(x)=x+6$ graphically and find the value of $f(x)$ when x is 15 using the graph or any other method, the student is said to be executing a flexibility framework for understanding. Therefore the incorporation of these two approaches (algebraic and object-representation) that give rise to the flexibility approach gives students a deep global understanding of the concept of functions since with this approach they will perform better in solving complex problems related to functions and their dynamic forms (Even and Bruckheimer, 2004).

Studies by Moschkovich et al. (2003) that required pre-service teachers to demonstrate the use of the flexible approach showed that the way some pre-service teachers responded to a set of given questions on compositions and inverses of functions suggested that with scaffolding, pre-service teachers were able to make meaningful mathematical connections. Based on research findings (e.g. Even and Bruckheimer, 2004; Carlson et al., 2004; Dubinsky and McDonald, 2001) it can be hypothesised that with reference to the composition and inverses of functions, a foundation student would be able to exhibit flexibility through the ability to conceptualize function(s) given in any representation. This can be done using the function view (object or process), the ability to represent composition and inverses of functions in a variety of forms, and to construct them from a given real-life or realistic mathematical problem. They should also be able to apply the four basic operations (+,-, x and /) to solve these problems.

Another aspect of flexibility that FP students can exhibit would be their ability to demonstrate understanding of composition and inverse functions by using appropriate representations (tabular, graphical, verbal, and algebraic) to show a composition or an inverse relationship.

Using the graph below

The graph of f(x) crosses the y axis at 2 and the x axis at -2.The graph of g(x) crosses the x axis at 2 and the y axis at 2. Given that both f(x) and g(x) are linear functions,

i. *Plot a graphical representation of g(x) and f(x) and use your graphs to find:*
   a) *The algebraic representations of g(x) and f(x).*
   b) *Represent f(x) in a tabular form using domain [-2; 3]?*

ii. *Use your understanding of the definition of the inverse of a function to draw the graph of the inverse of f(x)*

iii. *Formulate the composition function $f_o g(x)$ and draw it on the graph where f(x) and g(x) given above*

The question above is requiring the students to apply their flexibility and understanding of a function: (1) to translate a word problem in English into symbolic and graphical representations of functions. The starting point would be the representation of the function in word form into an object (the graph) and then work from the object to answer all the other questions that follow. Therefore students' dynamic understanding of the various forms in which a function exists will always help them to answer questions that are related to functions as they switch from one form of framework/ understanding of a function to another (see Figure 3.3**)**.

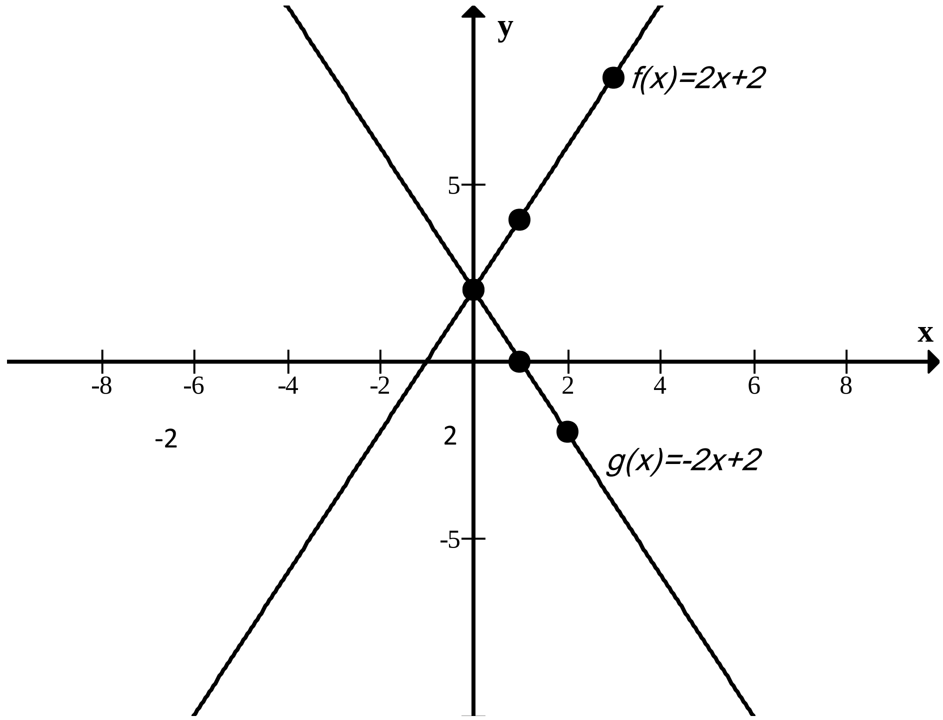


Figure 3.3: Sketch solution of the scenario above

b) $f(x) = -2x + 2$

x	-2	-1	0	1	2	3
f(x)	2	4	2	0	-2	-4

3.4.2 Structural-Operational Framework

Sfard (2004) developed a conceptual framework on how students perceive compositions and inverses of functions in mathematics, though this framework was somehow similar to the forthcoming APOS framework (see 3.4.2). Harel and Dubinsky (2004) state that mathematical concepts like functions, number and variables can be perceived from two perspectives which are complementary. The complementary perspectives are: as processes (operational view) and as objects (structural view). Harel and Dubinsky (2004) further define these two views as follows:

Operational conception: This occurs when an individual views a given notion as referring to a process rather than an object. By this they meant for example, that a function which is usually defined as a permanent static construct, (e.g. a set of ordered pairs), may also be perceived as a computational procedure. An operational conception of a function involves the viewing of a function as an algorithmic process action, rather than as a static object (Sfard, 2004). In this conception, a function is viewed as a computational process. Skemp (1971) defines this conception of a function as "a computational process or well defined method of moving from one system to another" (p. 246). Foundation Programme students can get to understand composition and inverse functions better through practical hands on experiences e.g. finding the inverse of function should be done practically for the student to understand the concept better. Sfard (2004) explains operational thinking as thinking in terms of doing, of operating on certain objects to get other objects. The researcher perceives operational conception of functions as equivalent to the actions and process levels of the APOS framework (see 3.4.3). As foundation students compose various functions trigonometric, logarithmic, exponential, etc. practically and developing the language and ability to explain such functions they are demonstrating the operational conception of a function.

A student who has an operational conception of the inverse and composition of a function has either a correspondence conception (e.g. Sfard, 2004) or a covariational conception (Carlson, et al., 2004).The correspondence conception is consistent with the traditional definition of a function as a set of ordered pairs (set-theoretic conception) with domain A and range B such that for every element in A there is only one corresponding element in B. The covariational conception defined by Carlson et al. (2004) refers to cognitive activities in coordinating two varying quantities while particular attention is paid to the ways they change with each other. The covariational conception is particularly useful in understanding rates of change in calculus (Carlson et al., 2004). Other researchers in mathematics education have referred covariational and correspondences conception of learning functions as cross-time and point-wise conception respectively (Kimani, 2008). For example, with regard to the functions $g(x) = 4 - x$, $f(x) = 2x$, a student who has a covariational conception of a composition function will be able to represent the function composition function $gf(x) = 4 - 2x$ and this will be represented as set of ordered pairs (see figure 3.4).

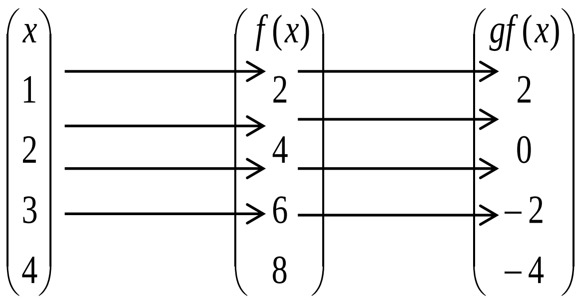


Figure 3.4: Representation of a composition function

As FP students demonstrate the above, they are showing their point-wise conception of a function, but if students can represent a function as a set of ordered pairs and are also able to represent the same function graphically and figure out that as *x* increases, *f(x)* decreases then they are demonstrating their understanding from both a covariational and correspondences conception of a composition function (see figure 3.5).

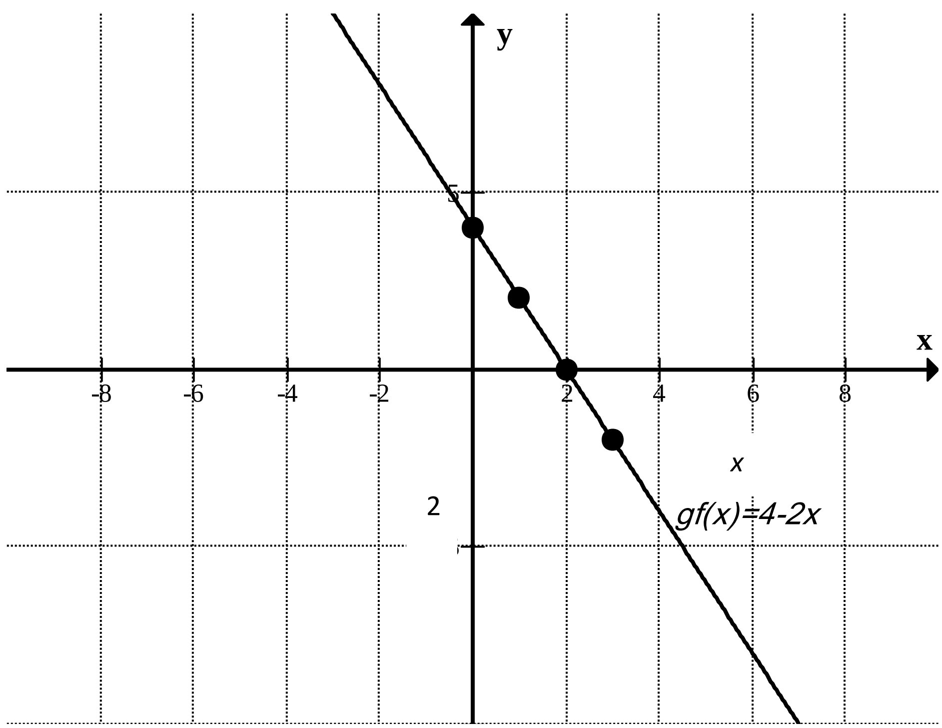


Figure 3.5: Diagram to demonstrate covariational conception of a composition function

A foundation student with a covariational (cross-time) conception of composition functions is able to describe the nature of the graph and to also explain how the two variables are related to each other (decreasing or increasing). In our present example *gf(x)* is decreasing with increasing *x* (see Figure 3.6). When FP students are able to demonstrate the above they will not face problems with handling the undergraduate module where they will be demonstrating their understanding of functions and their representations using differentiation and using calculus to find the areas bounded by functions using integration.

The structural conception occurs when a given notion is conceived as referring to an object (Sfard, 2004). Sfard further argues that the object is a metaphor which makes an abstract entity in the image of a physical thing. When functions are perceived as a set of ordered pairs rather than a computational procedure, they are perceived as an abstract object. Sfard further contends that a student who has a structural conception of functions is capable of referring to it as if it was a real thing – a static structure, existing somewhere in space and time. She adds that a student with this conception is able to recognize a function at a glance and is able to manipulate it without going into detail. Sfard (2004) argues that it's a "dynamic sequential and detailed" (p. 4). She further believes that a student with a structural view of a function does not need to act on a function to understand it. She also contends that this remarkable difference between the two views, "interpreting a notion as a process (operationally) implies regarding, it as a potential rather than actual entity, which comes into existence upon request in a sequence of actions" (Sfard, 2004, p. 4).

In light of this, Dubinsky and Wilson (2013) argue that structural thinking is advantageous over operational thinking because it is more integrative, economical, and more amenable to holistic treatment (parallel processing). From the researcher's perspective, this conception encompasses both the schema and object levels of the forthcoming APOS framework for understanding functions. FP students who possess structural thinking ability are able to create the necessary link between/among functions, manipulating them and perceiving an equation, algebra, functions and working for one common goal rather that perceiving functions from a situational/operational perspective and fail to create the most needed conceptual understanding that links functions with other related mathematical concepts. The FP lecturer should be able to create learning environments that foster the students' structural thinking. This could certainly be done by asking questions in such a way that the students will not give a yes or no answer, but instead will have to give justifications for their line of thought.

Sfard and Tomas (2008) argue that the two views - structural and operational - complement each other and are not mutually exclusive. The operational view develops first and then evolves into a structural view (Sfard, 2004). The structural conception develops from the operational conception through the processes of interiorization, condensation and reification. At the interiorization stage the student gets acquainted with the process which will eventually give rise to a new concept (e.g. algebraic manipulation which will turn into functions). From the researchers' understanding, this view is equivalent to interiorization of an action mentioned in the APOS framework discussed below by (Dubinsky et al., 2004). During the condensation stage, the student "squeezes" lengthy mathematical processes and is able to think about them in the form of a summary without necessarily putting down every detail of the processes (Hollebrands, 2004). From my own perspective, this stage is analogous to the coordination stage of the APOS framework elaborated earlier on (Confrey and Costa, 2006). All in all, if a student is able to look at the entire process of manipulation as an object then the process is said to be reified. I view this process as being similar to the encapsulation process of the previously discussed APOS theory.

Sfard (2004) argues that the ability to view a function from the process and object point is indispensable for deep understanding of mathematics. She further argues that operational and structural views of a function are inseparable facets of the same thing and exhibit a "duality rather than a dichotomy" (p. 9). The exhibition of the operational and structural views of functions is equally important for students to understand mathematics better. However, the context in which these are applied may also determine which perspective is more relevant and convenient. For example, the structural view of a function is a prerequisite for the effective understanding of transformation functions (Carlson et al., 2009; Dubinsky and McDonald, 2001; Dubinsky and Wilson, 2013). The understanding of composition and inverse functions requires both views since they complement each other (Sfard, 2004). Sfard further contends that once a student has become acquainted with the processes of manipulating compositions and inverses of functions and is now able to perform lengthy operations involving functions he/she is now in a position to handle their properties, which are more complex processes. Such a student has reified his/her functional operations into a structural view.

Bayazit and Gray (2004) argue that students who have attained an operational view of a function are more capable of handling function inverses in situations that do not involve operational formula. There is a greater acknowledgement among researchers who have used either the APOS framework or the operational/structural framework on the importance of moving flexibly between the operational (process) and the structural (object) views; however the students still face the dilemma of object-process duality of mathematical concepts (Gray and Tall, 2004).

Despite the well elaborated benefits obtained in the use of the APOS and the operational-structural frameworks of acquisition of mathematical knowledge through the object metaphor, Confrey and Costa, (2006) admit that presentations on these frameworks have been admirable and interesting. However, selecting mathematical objects as the critical metaphor in the theory may be retrogressive to the field of mathematical education. Confrey and Costa (2006) further argue that the shared belief among "reification theorists" that mathematical knowledge is acquired through transformation of processes into cognitive structures (objects), and that mathematical knowledge is a collection of knowledge in the knower's mental space assumes that the primary motivation to learn mathematics is to acquire objects, and that " the relationship between the knower and the known lies on the knower's ability to successfully move the objects into the mental space"(p.145). This claim suggests that mathematical educationists should not rely on the object-process metaphor but should however explore other richer relationships as well.

3.4.3 The Actions, Processes, Objects and Schemas (APOS) Framework

The Actions, Processes, Objects and Schemas (APOS) framework for understanding functions is a constructivist framework which is based on reflective abstraction (Harel & Dubinsky, 2002). Dubinsky and McDonald define the action component of the APOS framework as follows:

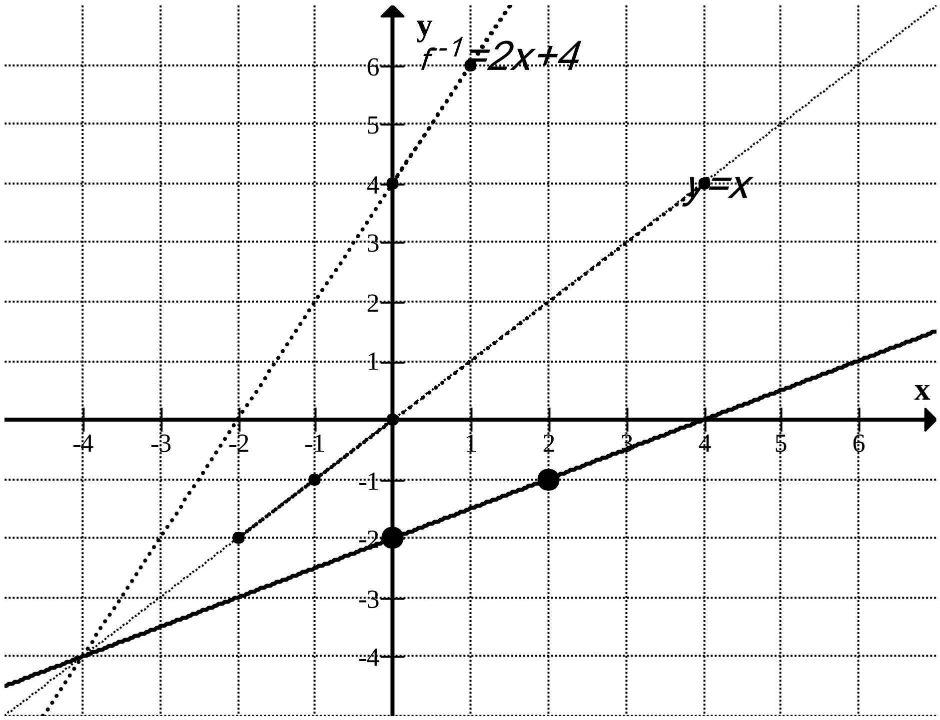
> "An individual's mathematical knowledge is her or his tendency to respond to perceived mathematical problem situations by reflecting on problems and their solution in a social context and by constructing or reconstructing mathematical actions, processes and objects and organising these in schemas to use in dealing with situations (p.6)."

Researchers belonging to the Research in SFP, in particular, (e.g. Moschkovic et al., 2003) adopted the APOS framework, which is a theory that arose in an attempt to understand reflexive abstraction that was introduced by Piaget. According to Piaget students' understanding of abstract concepts like functions can be achieved in four levels of reflexive abstraction which are: maturation, experience, equilibrium and social interaction. Piaget (1978) defines maturation as the students' ability to respond to the environment in an appropriate manner. This response is generally learned rather than instinctive. This process also encompasses being aware of the correct time and place to behave and knowing when to act, according to the circumstances and the culture of the society one lives in (Triandis, 2009). Maturation is the physical, intellectual, or emotional process of development. Maturation is often not quantifiable, and it too is mostly influenced by genetics. At the maturation level, students learn the step by step processes of performing an abstraction. For example, to find the inverse of the function $f : x \to 3x - 5$, the student at maturation level should first learn how to work with problems on subject of formula and should also know the step by step application of the BODMAS rule in abstract concepts, and performing substitution. At the level of experience students are experiencing something that, hopefully, results in a change in thinking, understanding, or behaviour afterwards. They begin to relate their experiences both formally and informally to what they are currently learning. A student who is trying to formulate the composition function of two linear functions will have to connect his experiences with substitution and simplification of algebraic expressions. The student will then use these old experiences to build on the new knowledge. This new knowledge which is built is based on the students' interaction with the society since, according to Vygotsky's (1978) social constructivist theory elaborated earlier (see 2.4), they do not learn in a vacuum. Equilibration is an attempt to bring about a state of equilibrium between the first three factors (old experiences, new knowledge and the environment) and the reality associated with one's external environment. This state must be present for cognitive development to take place. The equilibration processes involve assimilation and accommodation highlighted earlier (see 2.4).

APOS theory is therefore an extension of Piaget's theory elaborated above. APOS theory suggests that an individual's conception of functions [mathematical knowledge] is constructed in four main steps which are: Actions, Processes, Objects and Schemas (APOS) (Dubinsky and Weller, 2013). APOS framework has been very useful in understanding the students' learning of a broad range of topics in calculus, abstract algebra, statistics, discrete mathematics and other areas of undergraduate mathematics (Dubinsky and McDonald, 2001; Asiala and Carlson, 2006).

Dubinsky and McDonald (2001) state that a student with an action conception of a function always associates a function with a specific formula, rule or computation. Such a student will find *g(f(x))* by making substitutions of each occurrence of the variable *g(x)* with *f(x)* and will find the inverse of *y=f(x)* by swapping *y* with *x* and make *y* the subject of the formula, or making a reflection of the graph of *y=f(x)* over the line *y=x* (Harel and Dubinsky, 2002). Therefore foundation teaching of compositions and inverses of functions should not just foster the students' action conception, but should allow students to have a holistic conception of a function (action, process, object and schema). The classroom activities and teaching methods used in the Foundation Programme mathematics should be able to develop all the four conceptions of a function among foundation students.

Coming to the process component, Dubinsky et al. (2008) argue that through repeated actions and self-reflection, students are able to attain a process conception. The students can make internal mental constructions called a "process" which they can think of as performing the same action, but without the need of an external stimulus. Such students can think of the process of performing an abstraction without necessarily doing the abstraction practically and can also think of composing and reversing it with other processes. For example, if students are asked to draw the inverse of the function represented below of the function $f: x \rightarrow \frac{x-4}{2}$ without performing and algebraic manipulation



$$f(x)=\frac{x-4}{2}$$

Figure 3.6: Illustration of the process understanding of a function

The student with the process understanding of the function above will quickly remember that the inverse of the function *f(x)* is simply the reflection of *f(x)* along the line *y=x* and will quickly impose the inverse on the graph as expected (see Figure 3.6).

Therefore basing on these arguments, it can be concluded that students who possess a process conception of composition and inverses of functions have a more effective understanding of functions than those students with an action conception of functions because the process conception of a function allows students to view functions as a generalized input-output that defines a mapping of a set of input values to a specific set of output values (Dubinsky et al., 2008). Dubinsky et al. further posit that a student with the process conception of a composition function will think of a composition function as a coordination of two input processes where the input is processed by one function and its output is processed by the second function. They further suggest that such students will think of a function inverse as a mere reversal of mapping that defines a function from the output to the input. Dubinsky et al. (2004) further argue that a student with a process level of understanding inverses and compositions of functions can "reflect on, describe, or even reverse the steps involved in generating inverse and compositions of functions without necessarily performing them practically" (p.10). A student at this level of understanding of composition functions can describe the graph of *g (f(x))* where *g(x) =2x*-4 and *f(x)=x*^{2} as the process that shifts or translates every point on the graph of $2x^2-4$ four units to the right (see Figure 3.7 below).

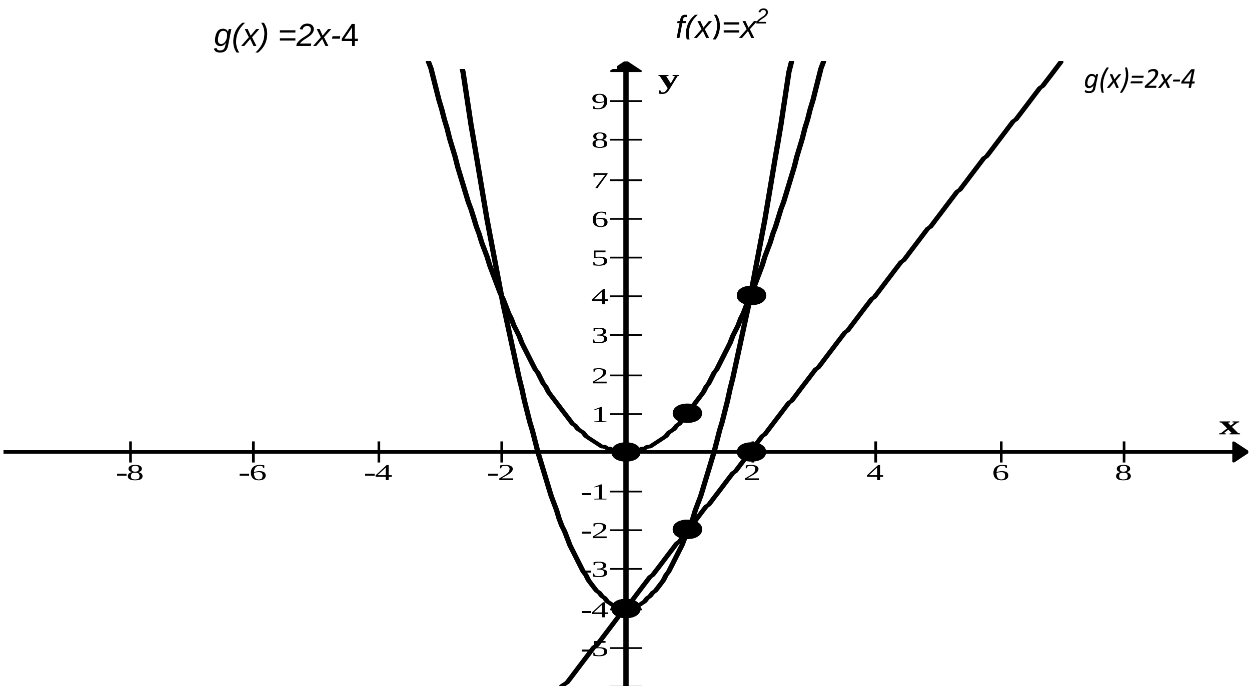


Figure 3.7: Process conception of a composition function

The ability of a student to conceptualize a particular process as an object that this process can be acted upon (e.g. adds the process to another, combine or compose it with another process), and can construct compositions, is said to be thinking of the process as an object (Asiala and Carlson, 2006). At this level the process has been encapsulated to an object. The student has acquired a static entity, and has an object view of the concept, e.g. a student can see a graph as linear, quadratic, or cubic, and can see the graph of one function as the inverse of another or a mapping as a composition function.

Finally, the student is able to build an understanding of the concept, which is a collection of actions, objects, processes and other schemas that are all required in the problem-solving environments (Dubinsky and McDonald, 2001; Cooney et al., 2011;

Dubinsky et al., 2008). In this framework, the student demonstrates coherence of the schema by showing an awareness of the scope of the schema (Harel and Dubinsky, 2002). A schema of a composition function is the same as what (Tall and Vinner, 1981; Kimani; 2008) refers to as the “concept image” of a composition function. Tall and Vinner (1981) argue that these schemas or concept images are constructed cyclically through interiorization, coordination, encapsulation, generalization and reversal. The researcher believes that this is the level of understanding at which students may be able to make such statements like “vertical shifts of functions are commutative” or ”composition functions are not commutative”. E.g. Given two functions $\mathrm{f(x)} = 4\mathrm{x} - 5$, $g(x) = x^2$ show that *fg* (x)≠gf(x) i.e. the composition functions are not commutative.

$$
\begin{aligned}
fg(x) &= f(x^2) \\
&= 4(x^2) - 5 \\
&= 4x^2 - 5 = P
\end{aligned}
$$

$$
\begin{aligned}
gf(x) &= g(4x-5) \\
&= (4x-5)^2 \\
&= 16x^2 - 40x + 25 = K
\end{aligned}
$$

$$P \neq K \Rightarrow fg(x) \neq gf(x)$$

The above illustration shows that composition functions are not commutative and that for the inverses of functions the schema includes an understanding of conditions for a function to be invertible, or that a function and its inverse swop their range and domain (Tall et al., 2000)

Researchers (e.g. Dubinsky and Wilson, 2013; Dubinsky et al., 2004; Moschkovich et al., 2003) using the APOS framework to construct students’ mathematical knowledge have suggested several issues with regard to the understanding of composition and inverses of functions. In the study of students’ understanding of composition functions in a computer environment, Even and Bruckheimer (2008) compared the performance of two groups of students - a calculus class taught using traditional methods and the other group was taught composition functions using computers. Both groups were given a pre-test and a post-test and the findings from the results suggested that students who were taught composition functions in a computer environment showed a greater capacity of reflective abstraction than those who used the traditional group. These results suggest that the use of computers to draw and create a meaning out of compositions and inverses of functions makes the learning more meaningful than when one has to do the graphing manually. A similar study was done by Asiala and Carlson (2006) to examine the effects of computers on students' understanding of compositions and inverses of functions. The results suggested that the computer environment helped students to develop a process conception of function and were able to perform specific mathematical tasks.

Basing on the findings above, Vidakovic (2006) designed a set of questions that were used in interviews (of student pairs) based on compositions and inverses of functions, to investigate how different students come to understand these concepts. The following genetic decomposition of the inverse of a function was deduced: Students have a process or object conception of a function and they coordinate two or more processes of a function to define the composition of two functions. Apart from that, they also make use of the previously constructed schema of a function to construct the inverse of a function. These findings lead to the development of an instructional sequence of functions that state that students' understanding of compositions and inverses functions was based on a process conception and not necessarily on the object conception of a function (Tall, 2003).

The APOS theoretical framework was also applied by Hollebrands (2004) when he studied the understanding of geometrical transformations of high school students taught through the Geometer's Sketchpad (dynamic mathematics software package). Hollebrands found out that students' understanding of geometric transformation seemed to progress from an action level of drawing transformations one point at a time to a schema level of being able to look at all the points on the plane as a possible domain of a transformation and thinking of properties of transformations. These findings were consistent with the findings of Carlson et al. (2004) who argue that the function conception ability of a student develops from an action view as "a single value as input or output at a time" into a process view where the student perceives a function as a transformation of entire spaces.

However, Tall (2003) claims that while there is a strong agreement among various mathematical educationist on the application of the APOS theoretical framework, it can be stated that cognitive operations preceed cognitive concepts, but in most cases the formation of a concept by a student does not follow a strict APOS framework which suggests that APOS is not a theory for everything, though it can be used as a template for everything.

3.4.4 Other frameworks for researching on functions

Apart from the APOS and the operational-structural frameworks for researching on students' understanding of functions, there are other frameworks which are also used by mathematics educationists. O'Callaghan (2008) used the conceptual framework of the function model. This model has four basic competencies which are: "modelling, interpreting, translating and reifying" (p. 24). O'Callaghan sees these competencies as accompanied by an associated set of procedural skills, such as:" "transformations and other procedures that allow students to be functional within a mathematical representational system" (p. 26). O'Callaghan's function model was based on Kaput's (1989) theory regarding sources in mathematics related to function concept. This study found out that there is a significant difference (O'Callaghan, 2008) in the way students in a traditional class perceived algebra when compared to those students in a Computer-Intensive Aided (CIA) class. Students in the traditional class had limited knowledge of one-to-one functions which they generally defined as equations, and had very little knowledge of their practical uses and applications, whereas CIA students had a precise definition of a function, that it is an operation that follows a certain pattern with a dependency rule that involves an input and an output value (O'Callaghan, 2008, p. 31)

It is worth noting that the research studies that have been cited so far have been in combination with a computer intensive environment. This investigation is in a PBL model applied to the teaching and learning of functions. None of the cited researches so far used a combination of the concepts. This gap in literature review justifies the need and relevance of the current research study in a bridging course for undergraduate mathematics.

3.5. PREVIOUS STUDIES ON UNDERTAKEN ON FUNCTIONS, COMPOSITION AND INVERSES

The concept of function has been investigated to a great extent in the field of mathematics education (e.g. Dubinsky, 2004; Vidakovic, 2007; Bayazit and Gray, 2004; Sfard, 2004; Dubinsky & Harel, 2004; Dubinsky & Wilson, 2013, Gulseren & Beyhan, 2014). These researchers point out the importance of the understanding and misconceptions of functions. Studies by Lucas (1996) found out that practicing and pre-service teachers have a limited understanding of functions. Trigueros and Martinez-Plannel (2010) assert that students still hold a primitive understanding of functions, and firmly rooted misconception which seems to have tied to historic views of functions which describe a function as a “formulaic” rule composed of a variable.

Although much research has been done on students’ understanding of functions; little attention has been given on transformation functions (functions of the form $f(x)=x^2+3$ which is a transformation of $g(x)=x^2$ by 3 units vertically upwards along the y axis)(Vidakovic, 2007; Boogni, 2009; Confrey and Costa, 2006). The same cannot be said about research on composition and inverses of functions (which are transformation functions too) in a Problem-Based Learning (PBL) approach, specifically in a Namibian SFP context. This limited research literature on the teaching of composition and inverses of one-one functions, specifically in a foundation program, shows that students still struggle with these concepts.

Therefore since functions are a central topic in the teaching and learning of mathematics from primary school to tertiary level in Namibian mathematics, the aim of this study was to improve on students’ understanding of inverses and composition of functions through the use of the PBL approach, based on the supplied evidence above, on how students struggle with these concepts which are usually taught using the lecture-based approach. These two concepts are normally taught separately through the lecture based approach in pre-calculus and the relationships between them are seldom discussed (Kimani, 2008). Despite this, when students get to pre-calculus in their first year, they are expected not only to know and to be able to apply these concepts precisely, but also to apply them in calculus problem solving and demonstrate flexibility in viewing and applying them. Gulseren and Beyhan (2014) argue that lack of students' knowledge of real life applications of functions causes defects in their realisation of the importance of order in number pairs.This, in turn, might cause diffficulties on their part in terms of understanding of functions or dfferent representations thereof (Dubinsky and Harel, 2004; Even, 2005; Asiala and Carlson, 2006; Dubinsky et al., 2004). In this regard real-life situations and teaching approaches like PBL that are compatiable with students own lives might be used to trigger meaning regarding the teaching and learning of compositions and inverses of functions.

Consider the problem below where composition and inverses of functions are being applied in real life.

You make a purchase at a local hardware store, but what you've bought is too big to take home in your car. For a small fee, you arrange to have the hardware store deliver your merchandise for you. You pay for your purchase, plus the sales tax, plus the fee. The tax is 9.5% and the fee is $30.

a. *Write a function t(x) for the total, after tax, on the purchase amount x.*
b. *Write another function f(x) for the total, including the delivery fee, on the purchase amount x.*
c. *Calculate and interpret i) (f o t)(x) and ii) (t o f)(x).*
d. *Which results in a lower cost to you? Assume tax, by law, is not to be charged on delivery fees.*
e. *Which composition function must then be used?*
f. *Show that* $\left(f \circ t\right)^{-1}(x) \neq \left(t \circ f\right)^{-1}(x)$
g. *Sketch the graph of* $y = f(x)$ *and that of* $y = \left(f^{-1}(x)\right)$. *How are the two graphs related?*

The teaching and learning of inverses and compositions of functions has been detached from reality to such an extent that when students are learning about them they cannot make connections with the real world. The problems above demonstrate how real-life situation can be used to demonstrate student flexibility in the teaching and learning of composition and inverses of functions, and how students can exhibit their understanding of these concepts in a PBL approach using real-life problems. Through the use of the above problems, the researcher anticipates that foundation students can be able to locate activities and events that happen in their daily lives that can be modelled by mathematical functions. Such problems also help to show whether students really understand that the inverse of a function is actually a reflection of the original function in the line $y=x$.

The PBL approach allows students not only to view functions as a concept detached from reality, but also to be able to make proper extraction of the mathematics in the daily problems they are confronted with (Even and Bruckheimer, 2004). This also leads to greater motivation once they are able to figure out the relevance of mathematics to real-life. Gulseren and Beyhan (2014) argue that the use of real-life problems fosters higher thinking abilities of the students, and improves on their problem solving abilities. They will also develop the proper metacognitive capacity that is needed when they are confronted with calculus which may also be modelling real-life events into differentiation and integration. Research has shown that traditional teaching approaches like the lecture method which dominates most tertiary institutions, do not achieve the goal of the Ministry of Education in Namibia; that of equipping students with fundamental understanding and proper problem-solving behaviours and the mathematical maturity that they need for success. Therefore the use of the PBL approach is a reformed teaching approach that is a paradigm shift from procedural teaching approaches to conceptual teaching approaches that foster the visual thinking ability of the student. On the issue of visualization, Dubinsky and Wilson (2013) state that:

> "The role of visual thinking is fundamental to the understanding of calculus that it is difficult to imagine a successful calculus course that does not emphasize the visual element of the subject" (p.136).

MoE (2003) calls for reduction of teaching only procedures but that teaching activities should be designed to foster critical and higher order visual thinking within the student. Since very few studies have specifically focused on evaluating students' experiences in the use of the PBL approach in teaching composition and inverses of one-to-one functions, the main thrust of this study is to find out what the students' experiences are with regard to the PBL teaching approach when applied to the teaching of compositions and inverses of one-to-one functions in a bridging course for undergraduate mathematics.

3.6. HYPOTHETICAL LEARNING TRAJECTORY (HLT)

Effective classroom teaching entails "meeting the students where they are" and helping them to build on what they know - the constructivist approach for teaching. However, lecturers in SFP still face challenges. These include making a precise distinction between what aspects of mathematics are important and which ones are not, how to make a diagnosis of what a student knows and does not know, how to build on the students' already existing knowledge - in what direction and in what ways (Clements and Sarama, 2009). Learning trajectories describe how students are likely to understand and develop their concept over time. The hypothetical learning trajectory gives a description of students' envisaged conceptual growth, from prior knowledge and informal ideas through intermediate understandings, to increasingly complex understandings over a period of time (Nicole, 2014).Lecturers in the SFP need to understand the learning trajectories for students so that they can prepare relevant materials that meets the cognitive demands for the students. Clements and Sarama (2009) define a hypothetical learning trajectory as a complex construction that includes

> "simultaneous consideration of mathematics goals, models of students' thinking, and teachers' and researchers' models of students' thinking, sequences of instructional tasks, and the interaction of these at a detailed level of analysis of processes"(p. 87).

With regard to compositions and inverses of functions, hypothetical learning trajectories describe the tentative learning path of a student from the less sophisticated to more sophisticated understandings, with great emphasis on what the student already has in the mind (prior knowledge). Nguyen (2009) defines an actual learning trajectory as a successive refinement that students exhibit (through instruction) as they progress from informal understandings towards "big ideas" (based on research on learning). Learning progressions/trajectories are described by Pelligrino, Chudowsky, and Glaser (2001) as

> "Successively more sophisticated ways of thinking about a topic that can follow one another as students learn about and investigate a topic" (p.214).

A hypothetical learning trajectory therefore is a teaching model - something a teacher conjectures as a way to make sense of where students are and where they might take them. This is hypothetical because the actual trajectory cannot be known until the actual teaching has taken place (Simon, 2005).In this study it is conjectured that the PBL approach will improve students' understanding of compositions and inverses of functions but the actual effect of the PBL will be seen when the teaching has taken place.

3.6.1 Underlying features of a hypothetical learning trajectory

Nicole (2014) describes four important features of a learning trajectory which the teacher needs to understand when teaching functions. The features are as follows (1) the teacher should emphasize on big ideas that will develop over time during his teaching, and (2) should also make a description of transitions from prior knowledge to more sophisticated understandings, (3) identification should be made to existing intermediate understandings and their contribution to conceptual growth and finally (4) a careful instruction, attending carefully to vertical instruction and to links across standards and topics. The teachers' understanding of the teaching and learning trajectories can improve their effectiveness in teaching, and can open up new ways of seeing students' mathematics, making the entire teaching endeavour more joyous and exciting since the students' mathematical reasoning becomes impressive and more delightful (Clements and Sarama, 2009).

With regard to the teaching and learning of compositions and inverses of functions, this study is consistent with Simon's (2005) perspective of hypothetical learning trajectories. The HTL for this study included three components which were: (a) specific learning goals for the Science Foundation mathematics students (b) a developmental trajectory or path through which foundation students develop to reach the targeted goal (c) and a well designed set of instructional activities that help bridging course for undergraduate mathematics students move along the intended path.

3.7 A CONCEPTUAL FRAMEWORK FOR UNDERSTANDING FUNCTIONS ADOPTED IN THE STUDY

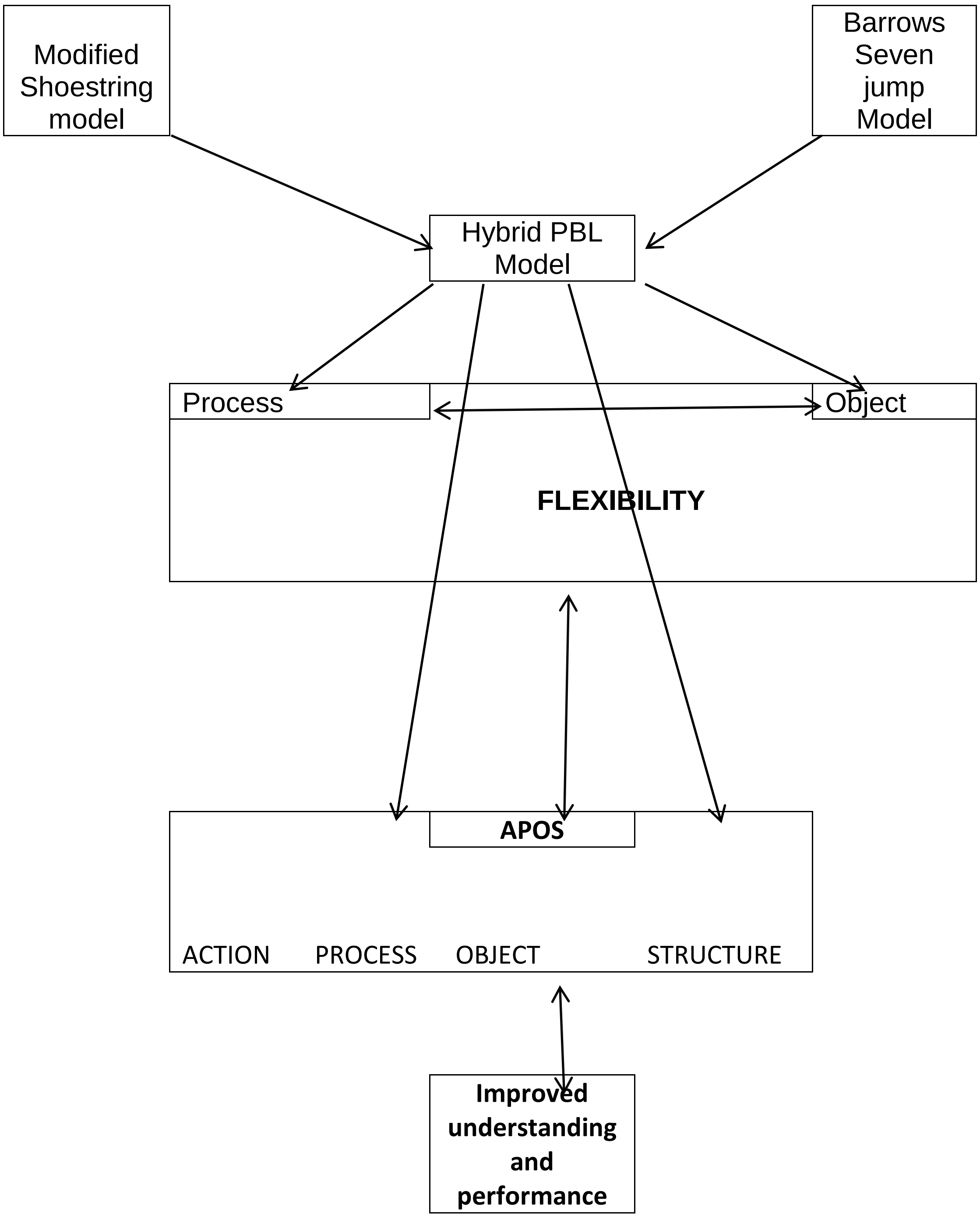


Figure 3.8: Conceptual framework for understanding functions adopted in the study: Adapted from Chirimbana (2014)

The study adopted the above conceptual framework for understanding functions presented in Figure 3.8. In Chapter 2 the researcher stated that the study adopted a hybrid model which was a combination of the Barrows Seven Jump model and the modified shoestring model for PBL (see 2.8.9). After a critical review of literature the researcher felt that the combination of the flexibility and the APOS framework for understanding functions were the most suitable frameworks for understanding the teaching of inverses and compositions functions. This was motivated by the fact that the two frameworks involve various ways through which foundation students mat perceive inverse and compositions of functions. These two frameworks have an aspect of knowing how to represent inverse and compositions of functions in various ways.

Figure 5.8 shows the diagrammatic representation of the adopted conceptual framework model which was formed out of the combination of the modified shoestring PBL model, the Barrows Seven Jump Models the APOS and the Flexibility frameworks for understanding functions. In the conceptual framework the hybrid model coming from the combination of the two became a suitable model to use to develop the foundation students understanding of inverse and compositions of functions from the flexibility framework involving the object and process, and the APOS framework involve the perceiving a function as an Action, Process, Object and Structure (see 3.4.2). Though the flexibility framework could be thought to be a subset of the APOS framework and that the students may utilise the APOS framework for understanding functions in such a way that they demonstrate flexibility, the two frameworks were treated as working for a common goal which was improved understanding of inverse and compositions of functions among the foundation students.

The hybrid model could help foundation students to understand inverse and compositions of functions from the APOS framework perspective or they could demonstrate their understanding of functions first by exhibiting their flexibility in the process and object presentations of functions (see 3.4.1), then exhibit the APOS framework. This refined model of conceptual framework was found to be suitable for a foundation programme like the one offered at Oshakati Campus.

3.8. SUMMARY

This chapter presented the background to and the evolution of functions in the teaching and learning of mathematics together with the various theoretical frameworks used by researchers to understand how students learn functions. It also presented the relevance of teaching and learning of functions in a FP together with the previous studies undertaken on the teaching functions including compositions and inverses. This chapter also presented the literature on HLT (a detailed conjectured HLT for the study will be presented in chapter 4). Finally this chapter presented the conceptual framework which was adopted in the study.

Chapter 4 will explore the methodological approaches used in this study. Research setting and timing will be explained. This chapter will also present issues of research ethics, validity and reliability issues, instrumentation sample and sampling procedures and clarification of data analysis

CHAPTER FOUR
RESEARCH METHODOLOGY

4.1. INTRODUCTION

This chapter presents a description of the methodologies (paradigms, research designs, research methods, instruments and sampling procedures) which were used in collecting and analysing the data for this study. It also gives the context and purpose of the study, research designs, development of the HLT, population and sample, sampling procedure, data collecting procedures, ethics, approach taken in the study, various phases of administering the research instruments and data analysis procedures.

4.2. RESEARCH DESIGN AND PARADIGMS

The study adopted a mixed-methods approach. The mixed methods paradigm attempts to get in the middle of the two other approaches (quantitative and qualitative), seeking to respect both by using both in a research study. Johnson and Onwuegbuzie (2004) state that mixed methods research is, generally speaking, an approach to knowledge (theory and practice) that attempts to consider multiple viewpoints, perspectives, positions, and standpoints (always including the standpoints of qualitative and quantitative research).

Creswell (2012) identifies two different types of mixed method designs as: convergent parallel design, where both qualitative and quantitative data is collected at the same time and the results are analysed, compared and interpreted concurrently. Then there is the explanatory sequential design where either the qualitative results are collected first and then analysed, and this builds up to the quantitative results or vice versa, with the purpose of interpreting the variations between the two. This study adopted the convergent parallel design since both the qualitative and quantitative data are collected, analysed and findings compared concurrently.

There are four worldviews within which a study can be located, and these are: the post positivist worldview where the researcher intends to verify an already existing theory, the pragmatic worldview where the researcher is dealing with real-world practice, the constructivist worldview and the participatory world view where the researcher intends to bring a change in a system based on his findings (Cresswell, 2012). This study adopted the pragmatic world view by virtue of the fact that the problem of poor performance is a real life problem and the mathematical problems which were studied were derived from real or realistic life scenarios.

The qualitative dimension of a research approach allows the researcher through the use of the interviews, conversations, field notes, recordings and photographs to observe, interpret and make sense of participants engagement/behaviour/ responses towards a phenomenon under consideration in a given natural setting (Denzin & Linkoln, 2005). In other words, the qualitative aspect of a research study places the individual at the centre as it focuses on investigating, discovering meaning and explaining particular phenomenon through the experiences and /or perspectives of the participants. In the case of the current study, focus group interviews and observations were used to capture the students' experiences of the PBL approach in the teaching and learning of inverses and compositions of function. In a qualitative research methodology the language of the subjects is important since the actual words of the subjects' experiences are critical in conveying the meanings and systems of the participants, which will ultimately become the findings of the research (Christensen, Johnson and Turner, 2010). In other words, whatever the participants say during focus group interviews is crucial since it necessitates the discovery of whatever is important and valued by the participants under study. However, the qualitative research methodology has the disadvantage that its findings cannot be generalised to other contexts.

Johnson and Onwuegbuzie (2004) encourage a broad interpretation of "methods" in mixed methods research to allow for "inclusion of issues and strategies surrounding methods of data collection (e.g. questionnaires, interviews, observations), methods of research (e.g. experiments, ethnography), and related philosophical issues (e.g., ontology, epistemology, axiology). Mixed methods research is a systematic integration of quantitative and qualitative methods in a single study for purposes of obtaining a fuller picture and deeper understanding of a phenomenon. Mixed methods can be integrated in such a way that qualitative and quantitative methods retain their original structures and procedures (pure form mixed methods) (Onwuegbuzie and Johnson, 2004). Alternatively, these two methods can be adapted, altered, or synthesized to fit the research and cost situations of the study (modified form mixed methods). Mixed methods research is also an attempt to legitimate the use of multiple approaches in answering research questions, rather than restricting or constraining researchers' choices (i.e., it rejects dogmatism). It is an expansive and creative form of research, not a limiting form of research (Cresswell, 2012). It is inclusive, pluralistic, and complementary, and it suggests that researchers take an eclectic approach to method selection and the thinking about and conduct of research.

The qualitative paradigm used a non-participant observation schedule (Appendix B) to evaluate student participation patterns in group activities (twice a week) and a focus group interview schedule (Appendices C and D) evaluating students' experiences. One sample PBL and one sample Comparison group was employed at the end of the teaching phase. A rubric (Appendix E) for assessing the group research projects (Appendix F) on composition and inverses of functions was also used. Cohen, Manion and Morrison (2007) state that qualitative research allows the researcher to gain an in depth understanding of the participants' experiences of the phenomenon. The researcher's decision to use the qualitative research methods for collecting data for inquiry stemmed from the nature of the research questions (see 1.5.), where the phrase "perceptions and experience" are key words. This implies that the researcher was also interested in subjective views of students on their experiences with the PBL approach in the teaching of inverses and compositions of functions. These subjective views and perceptions are influenced by the conditions created by the Foundation Programme which falls under the Faculty of Science of the University of Namibia.

The quantitative paradigm used a pre-test- post-test design with matched pairs for PBL (Experimental) group and lecture groups, then a survey design with a questionnaire with open and closed questions on students' experiences with the PBL or Lecture method. One self-developed Test 1 (see Appendix A) was used as an instrument for pre-test and post-test purposes. Christensen et al. (2010) state that the pre-test post-test design is suitable for all topics where the researcher seeks to establish a causal relationship and where it is possible to introduce and compare the stimulus (i.e. manipulate the independent variable which is the teaching method) at a specific time or to the specific groups of participants. The tests comprised of definitions of a function, representation of a function, formulating a composition and inverse of a function from a word problem and solving equations involving compositions and inverses of functions. This quantitative design had three distinct phases: preparation design phase, the teaching phase and post-test analysis phase. The use of a comparison group provides a baseline against which the experimental effects of the intervention may be evaluated (Tiwari A. , 2008). Thus, with a comparison group the researcher was reasonably confident that the major differences in the two groups were mainly due to the intervention (Cook and Campbell, 2003). The researcher was aware that implementing the non-equivalent control group design there was no certainty that the two groups were equivalent prior to the treatment which in this case was the PBL implementation. Table 4.1 below show a representation of the pre-test post-test design.

Group	**Pre-test**	**PBL Implementation Stage**	**Post-test**
E	O_1	X_1	O_2
C	O_1	X_2	O_2

Table 4.1: Untreated Comparison Group Design with Pretest and Posttest

In the Table above, X_1 represents the treatment rendered to the 40 students forming the experimental group. In the case of this study, rather than receiving no treatment, the students forming the comparison group were taught using Method X_2 (Lecture) considering that they enrolled for the FP to study and it would be further unethical to give them no instruction. Therefore with reference to the table above, it was the FP students in the PBL group that was represented with an X_1 and X_2 represented the comparison group. Even though pre-test and post-test designs are meant to render causal inferences as much as possible, it should however, be noted that the researcher cannot always control all the other relevant variables in real field setting considering the levels of complexities involved. As such, the validity of the experiment can still be threatened by other external variables which may be beyond the control of the researcher.

Christensen et al. (2010) assert that a quantitative research seeks to answer questions of how much and how many and is mainly concerned with establishing the extent to which variables relate to each other. Quantitative research takes the form of an experiment, quasi- experiment or non-experimental design. The non-experimental research design includes descriptive research that seeks to investigate situations, and relationships among variables without manipulation of the independent variables (Polit and Beck, 2004). It usually seeks to establish the extent of causal relationships between two or more variables, the strength of which can be tested using statistical methods (Christensen et al., 2010). In this study the researcher used experimental designs where he intended to establish the causal relationship which was the performance of students after being taught using a particular teaching method. This design enabled the researcher to use statistical methods to test the effect of these methods on the FP students' performances.

4.3. JUSTIFICATION OF RESEARCH METHODS AND INSTRUMENTS

4.3.1. Population and sampling procedures

The population for this study comprised of 140 students in the SFP of the University of Namibia based at Oshakati campus that were registered for a 1 year bridging course in the 2013 academic year. This study employed a mixture of both simple and stratified random sampling. Ahmed (2009) defines simple random sampling as a sampling procedure where each element has an equal probability of being selected among all the population units available. There are two types of Simple Random Sampling (SRS) strategies; the one with replacement and the other without replacement. Specifically, this study employed SRS without replacement, meaning that whatever student was selected for sampling it was not going to be returned into the population and stand a chance to be selected again from the population pool (Ahmed, 2009). The employment of simple random sampling was suitable for this study because randomization of participants eliminates the researcher's bias and favouritism to specific individuals in the study.

In the case of the employed stratified random sampling, Christensen et al. (2010) state that stratification is the process of dividing members of the population into homogeneous subgroups before sampling them. The strata should be mutually exclusive: every element in the population must be assigned to only one stratum. There were three FP classes - class 1, class 2 and class 3. Class one had 46 students and class 2 had 46 students while class three had 48 students. The different classes formed the strata from which the simple random sampling stated above was employed. Computer generated random numbers were used to select the different class samples, the 40 students forming the comparison group and 40 for the experimental group. In Class 1 and Class 2, 52 students were selected, 26 from each class, whereas in class 3, 28 students were selected. Randomization was suitable to make sure that all the three 2013 SFP classes forming the population for this study were fairly represented in the study.

4.3.2. Data collection procedures

The data for this study were collected through a questionnaire with open-ended and closed-ended questions, pre-post-tests; non-participant observation schedules, focus group interviews and a research project were employed in this study. This triangulation enables the researchers to compare the findings from the various information sources. Christensen et al.(2010) state that triangulation provides a better understanding of the phenomenon being investigated than if a single method of data collection is used. Details of how each of these data collection methods was administered in the study will be given next.

4.3.3. Administration of the pre-post-test

Pre-tests are given to students before a lesson or a unit to assess what they do (Kelly, 2009). Both the experimental and the conventional groups took the same pre-post-test (see Appendix A). The test was based on five HLT domains which were (D1) understanding the definition of a function (10%), (D2) representation of a function-14.3% (D3) ability to connect inverses and compositions of functions to real life-40%, (D4) ability to formulate inverses and compositions of functions from word problems-20%, (D5) manipulation of abstract mathematical problems involving inverses and compositions of functions-15.7%. The Table 4.2 below shows the HLT domain distribution by percentage:

Table 4.2: Pre-post-test HLT domain percentage distribution

Domain	Hypothetical Learning Trajectory Domain	Domain Total Marks	Domain(%)
D1	Understanding of the definition of a function.	7	10
D2	Representation of a function.	10	14.3
D3	Ability to connect inverses and compositions of functions to the real world.	28	40
D4	Ability to formulate the inverses and compositions of functions.	14	20
D5	Manipulation of abstract mathematical problems involving inverses and compositions of functions.	11	15.7
	Total	**70**	**100**

Since the two groups could not be fitted in one classroom to write the test, two writing sessions were prepared and the test was 90 minutes long, and was marked out of 70. The pre-test was meant to measure the FP students' prior knowledge about compositions and inverses of functions. Kelly (2009) states that pre-tests help to measure prior learning of the student and by comparing pre-tests and post-tests, the teacher can see what the students actually learned from the lessons that were developed. Pre-tests can also serve the unintended purpose of enlightening the students on what might be expected of them and will help them to focus on the main aspects on the key topics to be covered.

The results from the pre-post-tests helped the researcher to make changes to lessons to include further instruction and review. A pre-test was suitable in this study because the researcher wanted to find out a causal relationship, which was the students' performances after they have experienced the two teaching approaches in their different groups (Christensen et al., 2010).

In the case of this study, external validity could have been threatened. However, the participants for both groups were randomly selected from the population. Apart from that, suitable sample sizes were selected in order to minimise this threat. Having clearly identified the different threats to validity which might have influenced the findings for this study, and how they were minimised, the pre-test/post-test design with matched pairs was still found to be suitable to allow the researcher to investigate the causal relationships as specified. However, careful considerations to other various threats to validity will be considered when the findings for this study are being analysed.

Both the pre-post-tests administered in this study were tested for validity. In this study, content validity of the pre-post-test was determined. The content validity of the pre-post-test was determined by submitting the test sample to two colleagues who are mathematics specialists from another campus of the University of Namibia for them to see if the prepared test had all the necessary information needed to complete this study.

Any measurement tool that is set to perform a task of measuring has to be appropriate for that particular task. Measurement of students' perceptions and experiences with a specific teaching approach must be valid and reliable. The pre-post-test must measure with great precision what it is intended to measure. Therefore subjecting to reliability check will ensure the attainment of this academic expectation. Christensen et al. (2010) define the reliability of an instrument as the extent to which consistent measurements would be obtained if the same study was to be repeated. The pre-post-tests for this study were self-administered and uniform, and all participants in the experimental group and comparison groups were given the same amount of time to complete it. However, the researcher made clarifications to those students who did not understand what was expected of them. In order to ensure consistency in attribute measurement and to be quite sure that every change noted was observable and not due to the measurement process, the pre-post-test was tested for its reliability attribute. The test was subjected to a test-retest technique, based on the assumptions that the measured phenomenon did not change between any two testing times (Covington, 2000).

4.3.4. Tutorial observation protocol phase for both groups

During the implementation of the teaching phase, the PBL/lecture group had ten tutorial sessions; the researcher's colleague (specialist mathematics lecturer) observed two of the ten sessions and also two of the ten PBL/lecture lessons and completed observation sheets on one of the observations. The main observations for the tutorial and the PBL sessions will be combined and one report will be prepared on the main findings. Christensen et al. (2010) contends that the non-participant observation schedule had the advantage that the observer was not participating in either the PBL or Lecture group; therefore he would not have any favouritism to any of the groups in his observations. The main aspects for observations were on the various roles played by the lecturer and the students and the mode of feedback which was used during the lesson/tutorial sessions, how the various lessons and tutorial sessions were being introduced to the students. This observation schedule was relevant because perceptions and attitudes are observable attributes which are reflected through action (Tiwari, 1998). Therefore the use of an observation schedule enabled the researcher to collect more information that would triangulate the information collected from other data collection instruments about these attributes.

This study had some lesson and tutorial observation schedules which were used by a non-participant observer who was another specialist mathematics lecturer from one of the sister campuses. However, it has the disadvantage that the participant may have reactive effects once they note that they are being observed and change the way they should behave. Though reactive effects of the observed groups would have affected the validity of the study findings, the fact that the groups being studied did not know what exactly the focus of the observation was kept these effects to a minimum.

4.3.5. Administration of the post-test

At the end of the PBL implementation of the PBL/Lecture sessions, both groups wrote the same post-pre-test in order to evaluate the effect of the PBL and lecture approach used during the teaching phase. The post-test ascertained the students' experiences of the PBL and lecture approaches used during their implementation sessions. The detailed results of the post test will be presented in chapter 5.

4.3.6. Questionnaire administration

The experimental and the comparison groups were asked to complete a questionnaire with structured open-ended and closed questions at the end of the implementation of the PBL/lecture sessions. A questionnaire is simply a ‘tool’ for collecting and recording information about a particular issue of interest (Harry, 2013).The use of self-administered questionnaires as data collection instruments for this study has several advantages. Questionnaires can be completed by several participants in very little time and are highly targeted and also affordable and apart from that, the main cost incurred is the cost of printing and distributing. Milne (1999), states that questionnaires are more objective and cheaper to design in collecting a lot of information in little time. The first part of the questionnaire for this study provided the students’ demographic information. The second part elicited the students’ general perceptions on the two teaching methods. The third part of the questionnaire explored the students’ experiences with the two teaching methods. The questionnaires also tried to explore the various strategies students used to deal with inverses and compositions of functions.

Closed questions on the questionnaire were coded to enhance data analysis using Statistical Package for Social Sciences (SPSS). In order to ensure consistency in attribute measurement and to be quite sure that every change noted was observable and not due to the measurement process, the questionnaire was tested for its reliability attribute. The questionnaire was piloted on 30, 2013 students registered for the Basic Mathematics module studying at another campus of the University of Namibia. The questionnaire had 8 questions asking students on their experiences with the PBL and the lecture method, each of these questions was scored on a four point Likert scale (see Appendix G and H). The scale was made a four point because the researcher wanted them to finally group the responses into two groups only (agreeing and not agreeing). The issues of validity and reliability with regard to the questionnaire will be discussed in section 4.7.

The researcher acknowledges that the use of questionnaires as a data collection instrument has several disadvantages. One of the disadvantages of using the questionnaires is that the body language of the respondent cannot be observed as the respondent cannot be seen. The questions can be misunderstood by the respondents which may require a lot of time to clarify them (Harry, 2013). However, the administered pilot study helped the researcher to simplify any double-barrelled questions.

The content validity of the questionnaire was determined by submitting questionnaire samples to two colleagues who are specialist mathematics lecturers from the University of Namibia for them to see if the prepared questionnaire had all the necessary information needed to complete this study.

All the 8 questions in the questionnaire gave a Cronbach Alpha reliability value greater than 0.79 for both the experimental and comparison group. These results showed a high internal consistency (Cronbach, 1951). For comparing groups, Cronbach advises that the α value of 0.7 to 0.8 is regarded as satisfactory. But for the clinical application, much higher values of α are needed (Cronbach, 1951). This value shows that the instruments had a high level of reliability since the value was close to 1. Double-barrelled and other unclear questions, and their sequencing and arrangement were also reviewed in order to measure the right attribute in the final study (Hatcher, 1994).

4.3.7. Focus group interview phase for both PBL and lecture group

One of the main objectives of this study was to find out the FP students' perceptions with regard to the use of the PBL/lecture approach in the teaching and learning of inverses and compositions of functions. Kruger (2006) argues that focus groups are effective ways of collecting data, especially when one wants to find out how groups of people feel about a particular topic. The four basic steps to conducting a successful focus group according to Kruger (2006) were followed. The steps are: planning, recruiting, moderating, analysis and reporting. During the planning phase, the researcher had to decide on how many participants were to be used, the venue, and the amount of time to be taken during each focus group interview session, and the general logistics on how the discussion was going to be conducted. The recruiting phase involved sampling the participating students for the study, the moderating phase involved the process of mediating the group discussion during the group interview, the analysis phase involved the process of coding the data into specific themes and finally the reporting stage involved the interpretation of the data in line with the reviewed literature in order to answer the research questions of the study.

In order to make an evaluation of the students' experiences and elicit their perceptions about the use of the two approaches - the lecture and the PBL approach, this study conducted two focus group interviews (one for the experimental group and the other for the comparison group) and this was done at the end of the implementation of the teaching phase. 24 students were randomly selected (12 from the PBL and 12 from the comparison group) to take part in the groups' one-and-a-half hour focus group interview which was administered in the boardroom for Oshakati Campus. The use of a small group of 12 participants per group allowed the participants to have ample time to talk and deal with their topic in great depth. Elliot and Associates (2005) advise that the maximum number of individuals who should take part in a focus group discussion should be 12 for best results. The researchers further emphasise that a focus group discussion should only involve people who are knowledgeable on the topic to be discussed and the facilitator should be able to manage group dynamics well enough.

The discussion was divided into four sections i.e. Section A was on the students' general understanding of the teaching approach which was employed to them, Section B elicited a deep exploration of the students' feelings and perceptions about the teaching and learning of compositions and inverses of functions, then Section C focused on students' PBL/Lecture experiences in the teaching and learning of compositions and inverses of functions and, finally, Section D focussed on the students' recommendations based on their experiences (see Appendices C and D).

Krueger (2006), states that the use of the focus group interview allows the researcher to gain insight and understanding about a topic by hearing from the people. A detailed focus group interview guide is attached (see Appendices C and D) for both groups. However, Elliot and Associates(2005) acknowledge the one limitation of focus group discussions that they may waste a lot of time if not well directed. The researcher tried to have less control over the focus group discussion, which could result in loss of precious time and a dead end or irrelevant issues being discussed.

Harry (2013) states that focus groups usually provide immediate ideas for the improvement of particular products or concepts. Focus groups also help identify clear requirements of end-user as well as the other needs not addressed for example, in an educational setting. In addition, focus groups provide insights on the current position of the discussion at hand, as well as measuring the reaction of students with regard to the introduction of a new teaching approach (Creswell and Garrett, 2008)

However, there are several disadvantages that are associated with the use of a focus group discussion. Compared to individual interviews, Alexis (2012), points out that focus groups are not as efficient in covering maximum depth on a particular issue. A particular disadvantage of a focus group is the possibility that the members may not express their honest and personal opinions about the topic at hand. They may be hesitant to express their thoughts, especially when their thoughts oppose the views of another participant. Apart from that, interviewer can greatly impact the outcome of a focus group discussion. They may, intentionally or inadvertently, inject their personal biases into the participants' exchange of ideas. This can result in inaccurate results and can also lead focus group participants into reaching certain assumptions or conclusions about an idea or product. Out of fear in going against the opinion of the interviewer, or even out of fear of disappointing the moderator, participants may not disclose their true and honest opinions (Christensen et al., 2010). The researcher had to minimise control on the groups so that the participants could interact and express their opinions freely.

4.3.8. Research project evaluation rubric

Research comprises of creative work systematically undertaken with the purpose of increasing the stock of knowledge, including knowledge of man, culture and society, and the use of this knowledge stock to devise new applications. It is also used to establish facts, reaffirm the results of previous work, solve new or existing problems and support existing theorems (Cook and Campbell, 2003). The use of project-based learning that has gained greater foothold in the modern classroom settings in the western world and has made students become more engaged in learning when they have an opportunity to dig into complex, challenging, and sometimes messy problems that closely resemble real-life (Thomas, 2007). Project-Based instructional models involve students in investigations of compelling problems that culminate in authentic products and transform students into active roles like problem solvers, decision makers, investigators and documentarians. It is an answer to problems brought up by rote learning. Within the framework of educational enquiry, project work takes a more specific meaning to learning. Project-Based Learning is a recognised sub-set of version of PBL. The modification of the shoestring model into a hybrid model to suit the bridging course for undergraduate mathematics at Oshakati was also a methodological contribution of the study.

For the above reason, the participants for this study were required to design a project that was based on applications of compositions and inverses of functions in their various Problem-Based Learning groups (see Appendix F for the task or assignment). The students were requested to prepare a portfolio on their various findings. An assessment rubric for marking the project was prepared (see Appendix F).

Forooq (2011) acknowledges that the use of project work had the advantages that it helps to develop social norms and values among the students and that it also provided an invaluable opportunity for correlation of various elements of the subject content and for transfer for learning. However, the main disadvantage associated with project observation/evaluation protocol is that it cannot be used as a sole teaching strategy and cannot be used to cover the entire teaching content. Apart from that, project work is not economical in terms of cost and time. More so, supervision of project work requires special skills which are not always available to many classroom practitioners.

With regard to validity and reliability of the students’ group project rubric, this study observed Guba’s (1981) criteria for assessing validity and reliability attributes for project work. Guba (1981) proposes four criteria that he believes should be considered. The four criteria are: a) credibility (in preference to internal validity), b) transferability (in preference to external validity/generalizability), c) dependability (in preference to reliability), d) conformability (in preference to objectivity) (Guba, 1981). In order to ensure credibility, the researcher had to ensure that the adoption of the research methods was well established and this was done by pre-disclosing an assessment rubric (see Appendix E) to all the participating groups. With regard to the transferability aspect, Guba (1981) stresses that it is the responsibility of the investigator to ensure that sufficient contextual information about the project is provided beforehand to ensure trustworthiness of research projects. Such information was provided to the students by the researcher (see Appendix E and F). In light of dependability which will enhance reliability of a research project, Guba (1981) advises that the participants should present their project findings and should be questioned by other group members on whatever has come out of their projects. Based on their ability to answer on the questions from the audience, the researcher can establish both dependability and conformability of the produced product (see Appendix E) for the project presentation/assessment rubric).

4.3.9 Data analysis procedures

Quantitative data from the questionnaires and research projects for this study were analysed using the Statistical Packages for Social Sciences (SPSS) version 21. Means and variances for the students' obtained marks were calculated. Content analysed and thematic coding was used to analyse qualitative data from questionnaires, focus group interviews and the observation protocol. The group projects were assessed using a pre-set assessment rubric (see Appendix E). The group projects and presentations were assessed in terms of the process, product and oral presentations on the product. A wide range of data was collected for this study. The researcher made an attempt to represent the students' perceptions with regard to their PBL experiences in the teaching and learning of inverses and compositions of functions in different ways. The data were organised according to types. Data from the pre-post-test for both groups were analysed together using SPSS version 21, through descriptive statistics, representing it in frequency tables and graphs, analysing the means and standard deviations. Data collected from students' questionnaires were first grouped into qualitative and quantitative data. Qualitative data on students' perceptions with regard to the PBL teaching approach were analysed using semantic coding and themes. Data collected from the two focus group interviews were first transcribed and then analysed using content analysis and the development of emerging topological themes. Codes and categories were created to help build grounded theory for statistical hypothesis (Creswell and Garrett, 2008). The two sets of codes generated from the PBL and the Comparison group were compared and issues of differences among them included contradictions and surprises which were rectified accordingly. The researcher then triangulated the responses from the questionnaires, focus group interviews and observation scores, to see if there were any similarities or differences worth noting between the PBL group and the lecture method. Some of the research questions for this study were used as subheadings for analysing the qualitative data.

The results of the students' performance were analysed using descriptive statistics; the effect of the PBL approach on academic performance of the students was analysed using inferential statistics through the use of the *z* test. The Statistical Programme for Social Sciences (SPSS) version 21 was used to compute the descriptive statistics of the data findings and the *z* test. There were no notable constraints which might have influenced the results of this study.

The PBL group had a research project in which they had to design and present, which was based on inverses and compositions of functions. At the end of the project, students compiled a portfolio on their findings in their various groups and they were then awarded marks out of a 100 on the process, the product and the oral presentations made on the designed project according to the assessment rubric (see Appendix D).

4.5. ALIGNMENT OF RESEARCH INSTRUMENTS TO RESEARCH QUESTIONS AND HYPOTHESES

The research questions for the study were as follows: Table 4.3: Research questions and their coding

Table 4.3: Research questions of the study

Question	
Q1	What are the SFP students' perceptions with regard to the relevance of inverses and compositions of functions as concepts in a topic that determines their academic destinations?
Q2	What are the students' preferences of the presentation format for implementing the PBL approach for increased student learning outcomes in the teaching of inverses and composition of functions?
Q3	How do SFP students experience the PBL approach in the teaching and

	learning of inverses and composition of functions compared to those who are taught using the traditional lecture method?
Q4	How do the SFP students' performances on inverses and composition of functions as a result of their PBL experience compare with those of SFP students taught using the predominantly Lecture method at FP?

The four research questions of the study were trying find out students PBL in the teaching and leaching of compositions and inverses of functions with regard to the following aspects: Question (Q1) Students' perceptions, Implementation of the PBL approach(Q2), Performance impact (Q3) and (Q4). Therefore each of the administered instruments was seeking to address one or all the above questions.

Table 4. 4: Alignment of the instrument, research question, and statistical test and HLT domain and reference literature

Instrument	Hypothetical Learning Trajectory Domain	Question addressing the domain	Reference in literature	Research question being addressed	Statistical test
Pre-post Test	• Understanding of the definition of a function (D1).	8;2a;5	3.2.	**-PERFORMANCE IMPACT (Q4).**	Z test for the differences in two group means (for dependent and independent samples)
	• Representation of a function (D2).	2d; 8;3b;2d;2c;5	3.4.1.		
	• Ability to connect inverses and compositions of functions to the real world (D3).	1;3c;6;7;9	3.4.2.		
	• Ability to formulate the inverses and compositions of functions (D4).	10; 7;10	3.4.1; 3.4.2.		
	• Manipulation of abstract mathematical problems involving inverses and compositions of functions (D5).	10;4	3.4.1-3		
Observation	• Lesson commencement (O1)	Appendix I	2.7.2	**-EXPERIENCES (Q3).**	

Schedule	• Use of problems as starting points (O2).		2.7.6		
	• Nature of the problems given to the students (O3).		2.9.1	-IMPLIMENTATION (Q2).	
	• Student grouping (O3).		2.9.2		
	• Lecturer and students roles during the lesson (O4).		2.9.3		
	• Students' Mode of feedback (O5).		2.9.4	PERCEPTIONS(Q)	
			4.3.4		
Focus group Interview	• General teaching and learning approaches at FP(F1).			EXPERIENCES (Q3).	
	• Students' Perceptions with regard to the teaching of compositions and inverses of functions using the PBL approach (F2).	Appendices C and D		IMPLIMENTATION (Q2).	Z test for the differences in two population means for the comparison and the experimental group results.
	• Students' experiences with the PBL and Lecture approach in the teaching and learning of functions (F3).		4.3.7.	PERCEPTIONS(Q)	
	• Students' recommendation (F4).				
Research projects	• Process(D1)				
	• Product (D2)	Appendix F	2.2.2	PERFORMANCE IMPACT (Q4); EXPERIENCES (Q3)	Z test for the differences in two population means for the comparison and the experimental group results.
	• Oral presentation (D3)				
Questionnaire	• Tutorial helpfulness (B3).			-EXPERIENCES (Q3).	
	• Perceptions on effectiveness (B4).	2.5.2.			
	• Employment of a students centred approach during PBL implementation (B5).			-IMPLIMENTATION (Q2).	
	• Level of satisfaction with				

	problem solving (B6).	2.6.3.			
	• Sufficiency of the PBL resources (B7).			**- PERCEPTIONS(Q1)**	
	• The effect of using real-life examples (B8).				
	• Adequacy of the created conditions to necessitate PBL (B9).				

4.6. HYPOTHETICAL LEARNING TRAJECTORY (HLT)

4.6.1. The Foundation students' Learning Goals

Learning mathematics is a complex and multidimensional if not an inherently indeterminate process (Epson, 2011). Those involved in the teaching and learning of bridging course for undergraduate mathematics mathematics should understand all the different variables that affect the students' learning. Therefore, the main goal of research on students' teaching and learning is to simplify this complexity without sacrificing the ability for research to inform classroom teaching (Pelligrino et al., 2001). However, successful teaching can only be measured by the extent to which pre-set goals and objectives are achieved. The main goal for this study was to investigate the students' experiences with regard to the teaching and learning of compositions and inverses of functions taught using the PBL approach. Each teaching session was designed in such a way that it promoted adequate and important characteristics of productive reasoning and ability to conceptualize compositions and inverses of functions following the PBL approach.

4.6.2 Foundation students' learning activities on composition and inverses of functions including the pre and post-tests.

Each of the twenty teaching experiment sessions (ten teaching and ten tutorials) of the FP students included a number of ill-structured problems on compositions and inverses of functions following a PBL teaching approach. The activities allowed the FP students to think and to figure out practical applications and graphical representations of the inverse of a function and that it is a reflection of the graph of a function in the line y=x. With regard to compositions of functions students from both groups were expected to formulate and manipulate a composition function from a series of word problems. With regard to a function in general, some of the activities prompted the students to reflect on real-life events that resemble functions. The researcher presumed that these activities would increase students' understanding of inverses and compositions of functions and their applications in the real world. Each of the lessons lasted for two hours. The nature of the learning trajectory was as follows: In the first and the second lesson, students had to come up with a suitable definition of a function and then connect their definitions to real world, day-to-day events that can be related to functions. In the third and the fourth lesson the students listed the different types of functions they knew and their real world applications. In the fifth and sixth lesson students explored the inverses of one-to-one functions and their real world meanings. In the seventh and eighth lesson students explored the compositions of functions $gf(x)$ and $fg(x)$ and also proving that $fg(x) \neq gf(x)$. The ninth and the tenth lessons were preparations and deliberations on project work. In all these lessons students were using real world problems which they would translate into abstract mathematical expressions and create a meaning.

4.6.3 A developmental learning trajectory for foundation students

Reasoning through dynamic function situations has greatly facilitated the present day study on Hypothetical Learning Trajectory (Strom, 2008). This study was mainly influenced by the fact that compositions and inverses of functions remain some of the most difficult concepts in foundation mathematics and at tertiary level, yet these concepts are very relevant in laying a firm base for pre-calculus and calculus at university level. A HLT mainly focuses on specific domains of conceptual development in a student. In foundation mathematics, lecturers are perceived as agents who hypothesize learning trajectories for the purpose of planning tasks that connect foundation students' current thinking with possible future thinking activity. The researcher's five years of experience as a university foundation mathematics lecturer and 9 years' experience as an advanced level mathematics teacher helped to guide and direct all the activities on compositions and inverses of functions, taught using the PBL approach. The researcher also collaborated with two mathematics lecturers from sister campuses of UNAM on the final refinement of all the PBL activities on compositions and inverses of functions, which were used in this study. The students had to have a peculiar understanding of what a function is and the different types of functions that are used in mathematics together with their relevance to mathematics. The different PBL activities which were given were aligned towards fostering students' understanding of the various applications and functions in the real world. This helped to build a firm foundation on the foundation students' understanding of inverses of a function which were relying heavily on the students' ability to work with subject of formula and solving algebraic equation before they embark on composition functions which rely on the students' ability to substitute and simplify algebraic expressions. Therefore the sequencing of the concepts of functions was done based on the researcher's previous experiences with functions and areas of difficulties among the foundation students.

In a nutshell, the final PBL teaching experiment and HLT incorporated a variety of ideas and perspectives which were gathered from the reviewing of the relevant literature, the exploratory study, previously undertaken PBL research studies, and the researcher's own teaching experiences.

Figure 4.1 below gives a summary of the methods of enquiry for the HLT, which highlight the chronological phases of the PBL and cyclic patterns of conjecturing, testing, and analysing within each phase of implementation.

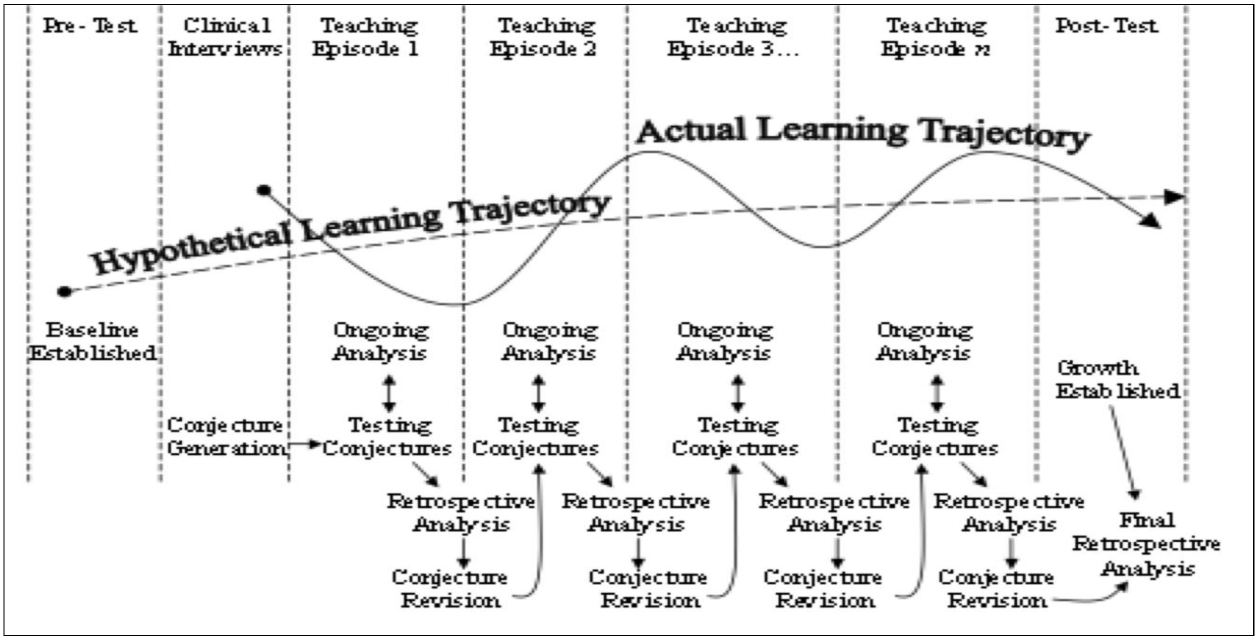


Figure 4.1: Methods of enquiry and cyclic processes of conjecturing, testing, analyzing and revising (Adapted from Steff and Thompson, 2005).

The HLT for this study is represented in the figure by the dotted curved line whose end points (after post-test) are generally higher than the initial point of implementing the PBL session (pre-test) in the teaching of compositions and inverses of functions. The actual learning trajectory, which is represented by the solid curved line, illustrates the fluctuating path of the students' learning in relation to the HLT initially conjectured. The fluctuation in the actual trajectory was a result of new concepts being introduced in each phase of the PBL implementation.

At the end of each PBL implementation episode, the mathematics lecturer from the sister campus who was also a non-participant observer retrospectively analysed the completed PBL experiments observation protocol in order to revise the initial PBL conjectures on compositions and inverses of functions. The cyclic process of conjecturing, testing, analysing and revising of the different types of functions, inverses and compositions of functions and real life applications of functions, taught using the PBL approach continued throughout the entire teaching duration. This was done to make an alignment of the previously conjectured learning trajectory of the teacher with the actual learning trajectory. Students were presenting their PBL findings in various groups as a way of giving feedback to the teacher, who would award them for their combined group effort in solving the problems.

4.6.4 The PBL classroom setting and sampling for the teaching experiment

The classroom setting for this study was situated in the fifth week of the second semester until the 12th week. Functions for the science bridging course for undergraduate mathematics are usually taught in the fifth week of the second semester. At the bridging course for undergraduate mathematics the teaching and learning of inverses and compositions of functions through the PBL approach focused on developing students' ability to connect compositions and inverses of functions to the realities of their everyday life and not as abstract concepts hanging in the air. Therefore, the 80 students randomly selected from the three FP classes (40 Experimental and 40 Comparison) received instruction in a combination of direct instruction, group discussions, whole class discussions, collaborative learning and class presentations.

These students met with the researcher for three month, two or three times a week for two hour sessions for three months. Out of the twenty two hour sessions, there were four PBL lessons and two observed tutorials. The observer was a specialist mathematics lecturer from the University of Namibia who had to complete an observation protocol of the tutorials. The classroom instruction was strictly a PBL style following the hybrid model of the modified shoestring model and Burrows Seven Jump Model (see 2.7.6; 2.7.2) and full description of the observation protocol (see 4.3.4) where a short and brief lecture about the problem is given and then students would be left to work on the PBL activities while the lecturer takes the role of a facilitator. The use of the shoestring model allows the facilitator to reactivate prior knowledge needed to grasp the basics of functions like mappings (definition, and types of functions, conditions for inevitability of a function, formulating an inverse of a function and formulating a composition function) since effective PBL implementation entails the reactivation of prior knowledge (Tan, 2000). During the implementation of the PBL session in the teaching of inverses and compositions of functions, a model of curriculum shift was employed in the PBL classroom in order to attain the expected HLT levels. Figure 4.9 shows the model of curriculum shift during the PBL sessions in the SFP during the PBL implementation.

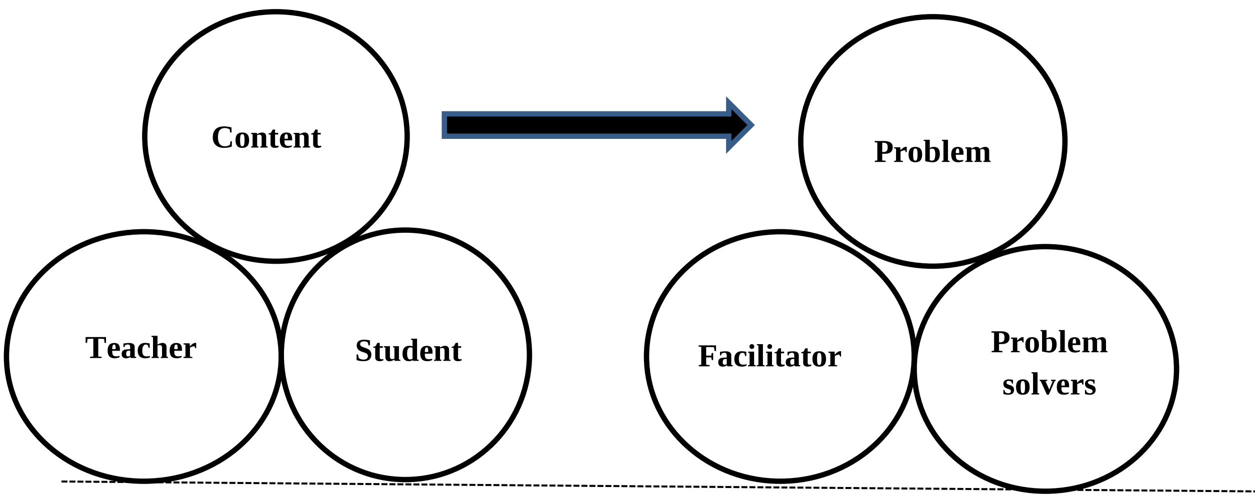


Figure 4.2: A model of curriculum shift during the PBL implementation.
(Adapted from Tan, 2000)

The Figure shows the didactic triangle on the shift of roles for the lecturer and those of the student and how this shifted during the course of this study. The lecturer assumed the role of a facilitator, and the students assumed the role of problem solvers while the content being taught on compositions and inverses of functions became the real world problem. This shift in roles was relevant for the FP students to reach the peak of their conjectured HLT level. In PBL classrooms, students are made autonomous thinkers in the real world and as such the assumption of the content into the problem and the students into the problem solvers, and the lecturer into the facilitator was in line with this goal for PBL and was a solution to rote learning (see 2.9.1; 2.9.2.)

In order for the students to reach the desired hypothetical learning trajectory levels as shown in Figure 4.8 above, the classroom environment must promote and emphasize a number of behaviours between the facilitator and the problem solvers as suggested by (Steff and Thompson, 2005).The following behaviours or classroom norms were agreed upon: (1) *speaking with meaning* - students were to speak meaningfully when engaging in PBL activities with their colleagues; (2) *exhibition of mathematical integrity* - group members and the facilitator were to base their conjectures on logical meaning foundation and not to pretend to understand when they did not; (3) *persistence in sense making* - the facilitator and the students were to persist in making sense not only of their own problem but also on the problems of other group members; (4) *respect for the learning process* - the facilitator and the group members were to allow each other to think critically and to also give each other opportunities to self-reflect and construct their own understanding during the problem solving process. These four rules of engagement continued to be reinforced throughout the entire PBL sessions and in the interview sessions of this study.

During the PBL implementation episodes students also utilized their calculators and other advanced level text books to enhance their understanding of inverses and compositions of functions taught using the PBL approach.

4.7. VALIDITY AND RELIABILITY

Christensen et al. (2010) define validity as the best approximation to the reality or falsity of a proposition. They further identify two main types of validity, namely internal and external validity. Internal validity refers to the approximation with which the researcher may make the inference that a relationship between two variables is causal while external validity refers to the extent to which the presumed causal relationship can be generalised or applied to the population under study.

With regard to this study, both internal and external validity may have been threatened by many factors. In light of internal validity, the study findings might have been affected by selection maturation which normally occurs when non-equivalent groups are growing at different rates in a common direction irrespective of the treatment. However, randomly selecting the participating groups for this study, this threat to validity was minimised by randomly selecting the participating groups. Another threat to validity in the face of internal validity was the Hawthorne effect or the observers' effect. This is a phenomenon whereby the control or comparison groups improve or modify their behaviour in response to the fact of change in their environment rather than in response to the nature of the change itself. In this study it was possible that some FP students could have been exposed to randomisation of sampling to eliminate this threat as such students were evenly distributed across the experimental and comparison groups. Thus, the plausibility of a local history should always be considered within the particular context of the study.

Another threat to internal validity is instrumentation. This refers to the situation when the intervals of a scale are not equal. Cook and Campbell (2003), advise that if the non-equivalent groups differ significantly from each other, there is a possibility of a floor or ceiling affecting one group more than the other. However, a close inspection of the extent of skewness of the frequency distributions within each group suggests whether instrumentation had an effect or not. Instrumentation can be reduced by raw data rescaling.

4.8. ETHICAL CONSIDERATIONS

Ethics refers to the part of human philosophy concerned with appropriate conduct and virtuous living (Given, 2009). Ethics involves the entire research process from the nature of the problem under investigation, reporting the theoretical framework underpinning the study, the research context, and data collection instruments and methods being utilised, the research participants involved and the procedures used to analyse the data (Creswell & Garrett, 2008). This study involved human subjects and as such, special precautions had to be taken to protect the rights of these human beings. Participants were students in the SFP which is a one year bridging course offered at the University of Namibia Oshakati Campus. They were therefore captive populations involved in an unequal relationship with the researcher who was also the lecturer for the population. In such a dependent relationship, these students may feel obliged to participate in this research study. They may fear that they may fail the course if they refuse to participate in the study conducted by their lecturer. Informed consent was obtained from the participants and they were also assured that their opinions were only to be used for academic purposes and their anonymity and confidentiality was guaranteed in all their participations. Specifically, this study observed three ethical principles, which were of particular relevance namely; the principle of beneficence, respect of human dignity and justice (Cresswell, 2012).

4.8.1. Ethical issues associated with participants and content

To uphold the principle of beneficence or “doing what is good”, (Roberts and Taylor, 2008) suggest that the main aim of the researcher should be to produce results which will be beneficiary to the individuals and the entire society at large. Apart from that, consideration for the potential for harm among the participants should also be observed. The study involved human participants who had to be put into two groups in which they would experience their different teaching strategies, therefore clear and careful elucidation of the risks and benefits of the study had to be made clear to the participants prior to the study. A clear measure of whether the risks involved would outweigh the benefits had to be made. The researcher obtained a written approval clearance from the University of Stellenbosch’s Ethical Clearance Committee (see Appendix P). Consideration had to be made so as not to disadvantage the comparison group in this research study. Therefore, they received their teaching in the form of the “standard” teaching methods of lectures and tutorials which were used in all the other subjects at the FP, which constituted method B rather than a classical control group.

The principle of respect for human dignity affirms the rights of students to self-determination, and the right to decide on whether to participate in the study or not, after full disclosure of the aim and purpose of the study (Good and Skeates, 2004). Full disclosure in this respect means that prospective participants should be informed of the identity of the researcher, the purpose and nature of the study, the right to participate and the right to withdraw anytime they wish to without any penalty, the responsibility of the researcher, and possible benefits of the study, measures to ensure privacy, anonymity and confidentiality. Students who participated in this study received full information about the purpose and objectives of the study so that they could make informed decisions about whether to participate or not (see Appendix L).

The principle of justice includes the participant’s right to fair treatment and privacy (Tiwari, 2008). This fair treatment should prevail before, during and after their participation in the research study. Furthermore, participants should be treated with respect and dignity and should always be free to ask the researcher for clarity on where they did not understand; and should they wish to withdraw from the study there should be non-prejudicial treatment.

A formal application to the University's Ethical Committee for clearance was made seven months before the study was undertaken. In making the application, a clear and detailed research proposal together with all the research instruments were submitted for ethical clearance. An informed consent form explaining the nature and purpose of the study was also completed and enclosed in the application (see Appendix L).

With regard to withdrawals, both groups were told that they were free to withdraw from the study should they feel they did not want to continue participating in the study. The participants then signed consent forms.

4.8.2. Ethical issues associated with research questions

Clear explanations of the purpose of the research were made and no changes were made on the original research questions after the research was completed.

4.8.3. Ethical issues associated with data analysis and reporting

Students were assigned alphabetical codes as their names for purposes of follow up only. The students then met in their usual mathematics classroom and wrote the pre-test in their different groups since they could not be fitted in one classroom. Participants were assured that their information would be treated with the strictest confidence. Data were captured on a personal computer which was password protected. Pseudonyms were used for the research participants and they were assured of anonymity. Participants were also informed that they were free to withdraw should they have wished to do so.

4.9 RESEARCH PROCESS ADOPTED IN THIS STUDY

Although the researcher fully believed that critical thinking could be enhanced by the implementation of Problem-Based Learning, he was quite mindful of the fact that foundation mathematics students were not going to accept this learning strategy with ease since it was the first time for most of them to experience it. Therefore effects of academic background from high school and socialization could make them wonder what sort of a teaching/learning strategy PBL was, which did not seem to tally with their previously experienced conventional approaches.

4.9.1 Phase 1: Pilot study

The research instruments were piloted to 30 randomly selected students from one of the campuses of the University of Namibia who's academic and environmental settings resembled those where the study was undertaken. The group of 30 was pre-exposed to a two week PBL sessions. Vague and double barrelled questions in the pre-post-test and questionnaires were restructured, based on the results of the pilot study. The pilot study group were not from the science bridging course for undergraduate mathematics though their academic setting resembles that of the FP. The 30 students were gathered in one of the classrooms and the researcher explained to them the purpose of their gathering (to complete pilot study questionnaires and pre-and post-test for a forthcoming study). The entire process took about one and a half hours. Details of the results of the pilot study will be discussed in Chapter 5.

4.9.2 Phase 2: PBL workshop with the PBL and comparison group students.

The researcher held a two hour workshop with the PBL and comparison group in separate sessions (Appendix K). The main aim of the workshop was to explain to the participants the various features of the PBL, a new academic approach they were going to experience, and the roles of the lecture group during the teaching sessions. These workshops was important to both the researcher and the students in that each student had to know the various roles they were going to play during the course of the lecture and PBL implementation. In these various groups some members were going to be writing what the group members had agreed upon while others were responsible for coordinating the group proceedings. In this workshop students who had some misconceptions were also assisted. It was during the workshop that the researcher discussed and agreed with the students on the behaviours to be exhibited during the PBL sessions as highlighted earlier on.

4.9.3 Phase 3: Administration of the pre-test to both groups

In this phase both the experimental and the comparison groups were administered into the pre-test (see Appendix A).

4.9.4 Phase 4: PBL and lecture implementation and lesson observation phase for the two groups

As FP students worked with problem situations, they were expected to practice and exercise critical thinking and were also expected to develop an inquiring mind in solving problems on inverses and compositions of functions in a PBL approach after allocating themselves into groups of 4 or 5. The PBL model which was adopted in this study was a hybrid model of a modified shoestring model and Barrows seven jump approach pioneered by Savin-Badin and Mayor (2004) where a mixture of PBL sessions and lectures are incorporated during the PBL sessions (see 2.7.6). A brief explanation about a problem was given by the researcher and then a PBL session would follow where students had to apply their knowledge in solving real-life problems based on definitions of a function, types of functions, inverses and compositions of functions. Students would explore the problems in their various groups using Barrows seven jump approach for problem solving as elaborated in 2.7.2, and they would make presentations of their group findings on the questions allocated to them. Students were given opportunities to practice dialectical thinking during group tutorials where they had to explain and justify their own positions and explore opposing points of views (Tiwari, 2008). The researcher hoped that as foundation students played this kind of “mental gymnastics”, they would develop a broader perspective in their understanding of inverses and compositions of functions. The use of open-ended problems during the PBL sessions would give the students an opportunity to explore and critically analyse the PBL questions and apply their problem solving skills.

During the PBL sessions, the researcher played the role of a facilitator while the students were the problem solvers and the problem solving sessions transpired quite naturally and students made their presentations under a free atmosphere.

4.9.5 Phase 5: Tutorial and lesson observation protocols for both groups

In this phase two tutorials and two lessons were observed for both the comparison and the experimental groups (see Appendix B and I) for the observation protocol sheet and the tutorial questions. Similarities and differences between the tutorials and between the PBL sessions will be compiled on two reports i.e. tutorials and PBL sessions.

4.9.6 Phase 6: Administration of the post-test to both groups

At the end of the PBL implementation students from both groups were administered into a post-test which was the same test they wrote at as a pre-test (see Appendix A).

4.9.7 Phase 7: Administration of the questionnaire to both groups

In this phase the two groups were asked to complete a self-administered questionnaire with close ended and open ended questions (see Appendix G and H).

4.9.8 Phase 8: Administration of the focus group interviews for both groups.

The two groups were then administered into two separate focus group interviews (See Appendix C and D) for the interview guide.

4.9.9 Phase 9: Presentation of the research projects.

This was the last phase of the PBL implementation where students were presenting their group projects (see Appendix F and E) for the project question and the assessment rubric.

4.10. SUMMARY

This chapter discussed the research designs which were administered in this study, and then it gave a hypothetical teaching methodology for the study. In addition to that, issues of populations and sampling procedures were also elaborated in this chapter, and then procedures and the different instruments for data collection were also discussed together with issues of validity and reliability for each instrument administered. Apart from that, this chapter also presented the ethical issues which were considered before the study was undertaken. Finally, it highlighted the different data analysis procedures which were employed for each set of data collected in the study.

Chapter 5 will present the data from the various data collection procedures of the research study, of the implementation of a PBL approach in the teaching inverses and compositions of functions.

CHAPTER FIVE:

INTERPRETATION OF DATA AND RESULTS

5.0 INTRODUCTION

This chapter presents the findings and interpretations thereof. The findings for this study will be presented as follows: analysis of the pre-test results for both the experimental and the comparison groups. An analysis of the results for the observation protocols for one lesson and one tutorial session for each group and will then give an overall comparative analysis for both groups on the observations made. This chapter will also present the results from the post-test followed by a comparative analysis of the two groups. It will also present a hypothesis test on the effect of the PBL approach. The chapter will then present the results of the group project presentations together with their comparative analysis.

5.1 ANALYSIS OF THE QUESTIONNAIRES AND THE PRE-POST-TEST RESULTS FOR THE STUDY

5.1.1 Demographic information for the comparison and experimental group

The comparison group had 40 participants comprising of 15 (37.5%) males and 25 (62.5%) females. The ages of the comparison group members ranged from less than 17 to more than 20. Figure 5.1 below shows the age distribution of the comparison group participants.

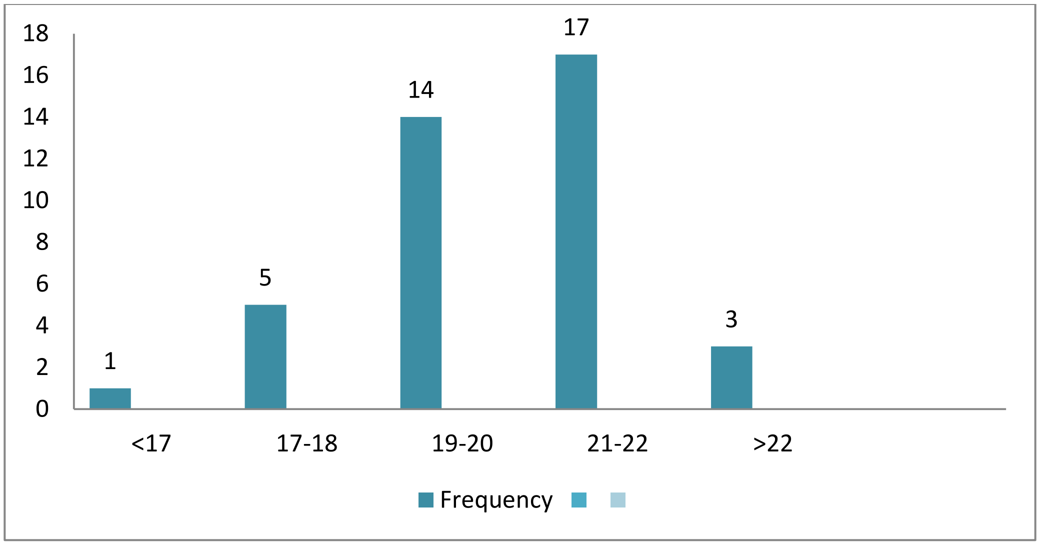


Figure 5.1: Age distribution of the comparison group participants

Figure 5.1 shows that one (2.5%) of the participants was below 17 years, whereas 5 (12.5%) were aged between 17 and 18, 14 (35.0%) were aged between 19 and 20 years, 17 (42.5%) were aged between 21-22 years and the last age group which had 3 (7.5%) participants above 22 years. The experimental group had 12 (30.0%) males who participated in the study, and 28 (70.0%) females who also participated in the study. Their age distribution is represented in Figure 5.2 below.

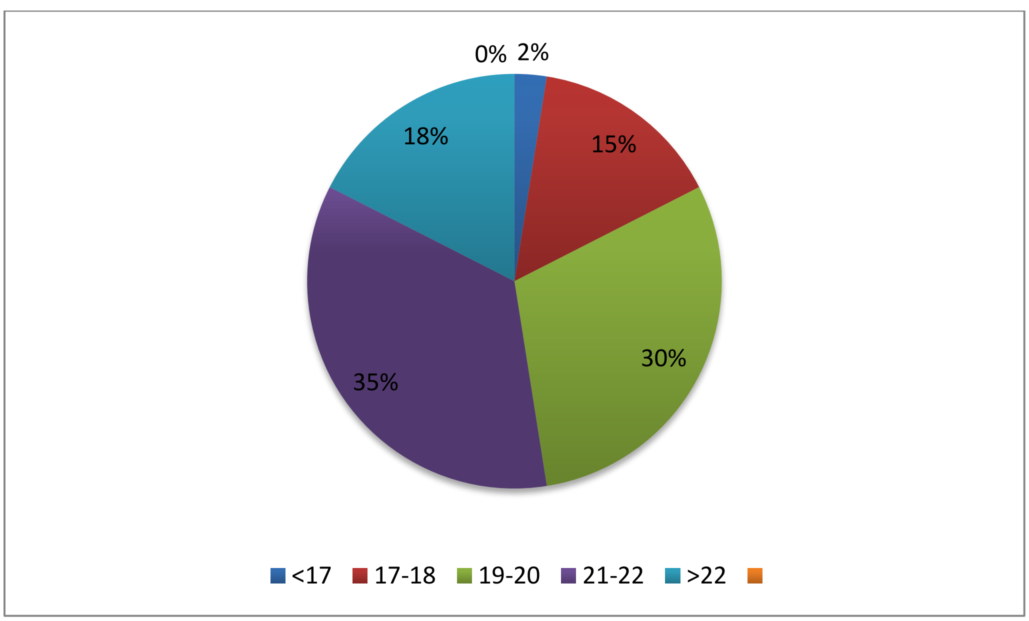


Figure 5. 2: Age distribution of the experimental group participants

Figure 5.2 shows that out of the 40 participants of the experimental group, one (2.5%) was less than 17 years, whereas six (15.0%) were aged between 17 and 18 years, 12 (30.0%) were aged between 19 and 20 years, 14 (35.0%) were aged between 21 and 22 years and seven (17.5%) were more than 23 years. The students who participated in this study were randomly assigned into these groups using a table of random numbers. That is probably the reason why the age distributions are approximately normally distributed.

5.1.2. Comparative analysis of the demographic information for the comparison and experimental groups

Both the comparison and experimental groups had only one participant who was less than 17 years. The modal age group for both groups was 21-22 years with the comparison group frequency outweighing the experimental group by three. The combined analysis of the demographic information of the two groups is summarized in the Table 5.1 below.

Table 5.1: Comparative analysis of the demographic information for the comparison and experimental group

Group	**Males**	**Female**	**<17**	**17-18**	**19-20**	**21-22**	**>22**
Comparison	15	25	1	5	14	17	3
Experimental	12	28	1	6	12	14	7
Total	**27**	**53**	**2**	**11**	**26**	**31**	**10**

The results in Table 5.1 above show that the proportion of males who participated in the study was 33.75% while that of females was 66.25%. These differences are in line with the proportions of males who were enrolled in the bridging course for undergraduate mathematics in the 2013 academic year which was 55 out of 140 students. It can be seen that female percentage outweighed the male percentage but the number the males in the comparison group outweighed that of the experimental group by three. The bridging course for undergraduate mathematics tends to attract more females than males probably because more males qualify to go for tertiary education while females do not.

5.2. ANALYSIS OF THE PRE-TEST RESULTS FOR THE EXPERIMENTAL AND COMPARISON GROUP

The comparison group and experimental group wrote a pre-test which was to determine participants' entry knowledge within and between the groups to benchmark effect of the PBL approach. As shown in Appendix A the pre-post-tests comprised of questions based on students' understanding of the following domains: understanding of the definition of a function (D1), representation of a function (D2), ability to connect inverses and compositions of functions to the real world (D3), ability to formulate the inverses and compositions of functions (D4), Manipulation of abstract mathematical problems involving inverses and compositions of functions (D5). The pre-test was marked out of 70. Table 5.2 below shows a summary of results of the two groups.

Table 5.2: Comparison group's performance by HTL domain in the pre-test

Domain	**Hypothetical Learning Trajectory Domain**	**Domain Total Marks**	**% weight**	**total marks obtained by 40 students**	**Total possible marks by 40 students**	**pre-test % pass for comparison group**
D1	Understanding of the definition of a function.	7	10.0	121	280	43.2
D2	Representation of a function.	10	14.3	198	400	49.5
D3	Ability to connect inverses and compositions of functions to the real world.	28	40.0	528	1120	47.1
D4	Ability to formulate the inverses and compositions of functions.	14	20.0	129	560	23.0
D5	Manipulation of abstract mathematical problems involving inverses and compositions of functions.	11	15.7	221	440	50.2
	Total	**70**	**100**	**1197**	**2800**	**42.8**

Table 5.3 shows that at pre-test the comparison group domain D2 which had 10% of the items had an overall pass rate of 43%, D2 which had 14.3% of the items had an overall pass rate of 40.5%, D3 which had 40% of the items had a pass rate of 47.1%, domain D4 which had 20% of the items had a pass rate of 23.0% and finally domain D5 which had 15.7% of the items had an overall pass rate of 50.2% at pre-test and an overall pass rate of 40.46% of all the domains. Table 5.3 shows a summary of the pre-test results.

Table 5.3: Summary for the pre-test results for both groups (N=40)

Group	Range	Minimum	Maximum	Mean Percentage		Std. Deviation	Variance
					Std. Error		
Comparison	38.57	0	38.6	30.52	0.839	7.578	57.426
Experimental	40.00	10.0	50.0	32.3	1.262	11.4	129.96

Table 5.3 shows that the comparison group had a mean score of 30.52 with a standard deviation of 7.578 while the experimental group had a mean score of 32.2 with a standard deviation of 11.40. The mean difference between the two groups was 1.25 in favour of the experimental group. The range of the experimental group was 40 while that of the comparison group was 38.5. A range difference of approximately 1% in favour of the experimental group was recorded. At a glance these results may suggest that the experimental group performed significantly better than the comparison group. The researcher performed a two tailed *z* hypothesis test for the differences of the means of the two groups to find out if there was a significant difference in the performances. The results of the two- tailed *z* test showed that ($Z_{\mathbf{Calculated}}$=0.825; $Z_{\mathbf{Standard}}$=1.96; α=0.05) and accepted the null hypothesis which stated that (H_0:μ_E-

μ_C=0). These results show that there was no significant difference in the means of the comparison and experimental groups before the treatment was applied.

5.3. COMPARISON GROUP PRE-TEST RESULTS

5.3.1. Definition of a function (D1)

Most students in this group gave different definitions of inverses and compositions of functions:

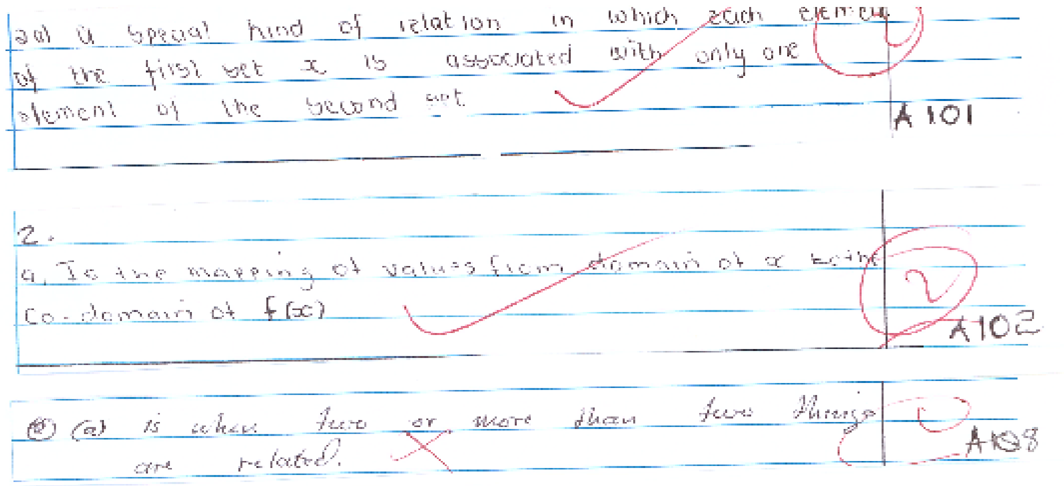


Figure 5.3: Definition of a function

-It is a relation between input and a set of permissible output-A140

5.3.2. Representation of a function (D2)

In this domain students were required to show using examples their understanding of a function and its representation and were asked to use any relevant ways they know which can be used to represent inverses and compositions of functions. With regard to the representation of a function, students in the comparison group gave the following responses:

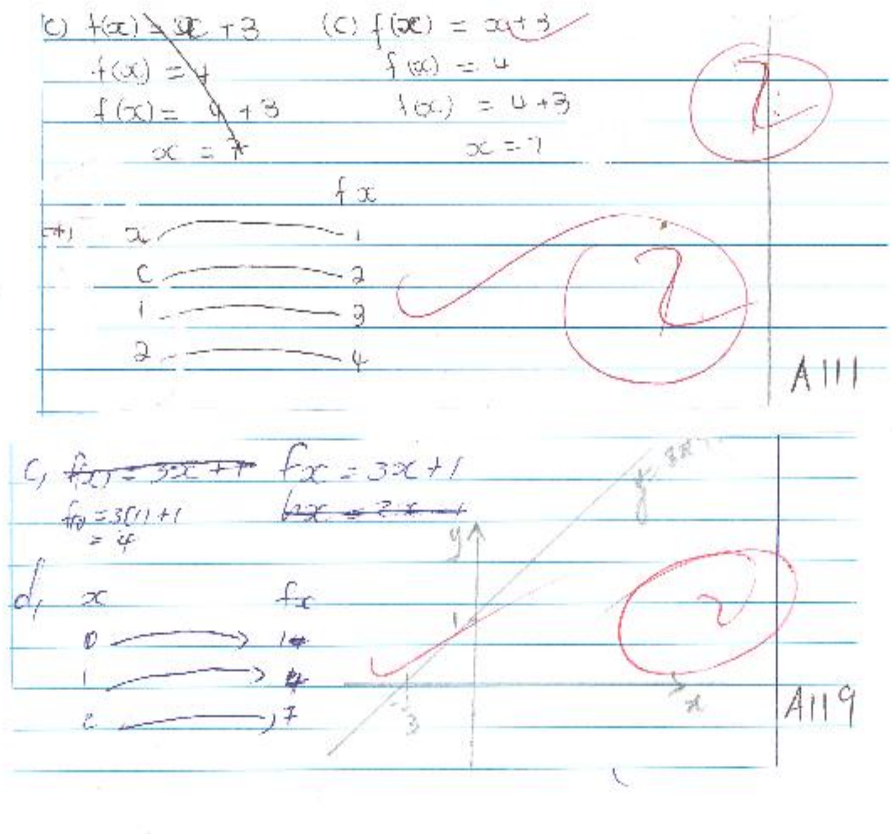


Figure 5 4: Comparison group’s representation of a function

-No Idea A114

5.3.3. Connection of inverses and compositions of functions to the real world (D3)

In this category students were assessed on their ability to come up with relevant examples of inverses and compositions of functions in the real world and using the concept of inverses and compositions of functions to solve real life problems on given scenarios. *As you get into a taxicab in Oshakati, the meter will immediately read N$3.30; this is the “drop” charge made when the taximeter is activated. After that initial fee, the taximeter will add $2.40 for each kilometre (K) the taxi drives.*

a) Write down a function of the total charge (C) payable for a distance of K km.

b) How many km did john travel if he paid N$100=00?

-No idea- A129

- a) C=3.30+2.40(k)

b) No idea -A135

-I don't know

5.3.4. Formulation of inverses and compositions of inverses and compositions of functions (D4)

The comparison group students were asked to formulate and graphically interpret inverses and compositions of functions. They were given a graph of a function which was identified as *y=f(x)* and were asked to draw the graph of the inverse of the graph. On this aspect the researcher was testing the students' knowledge of the property that the graph of the inverse of a function is the reflection of the function's graph along the line *y=x.*

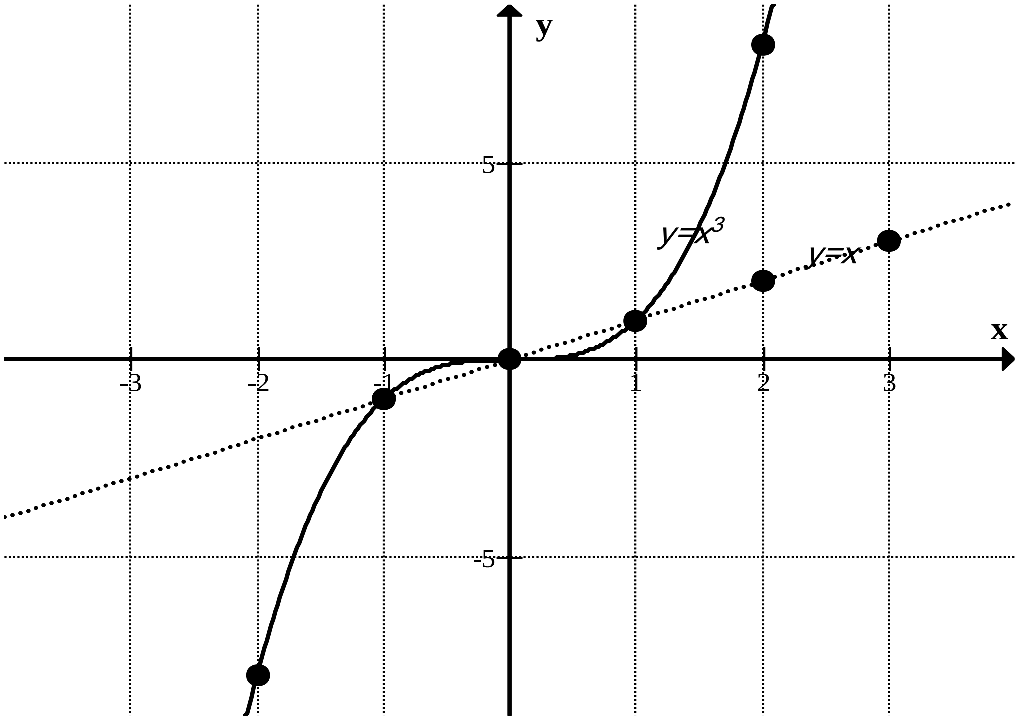


Figure 5.5: Function whose inverse was to be superimposed

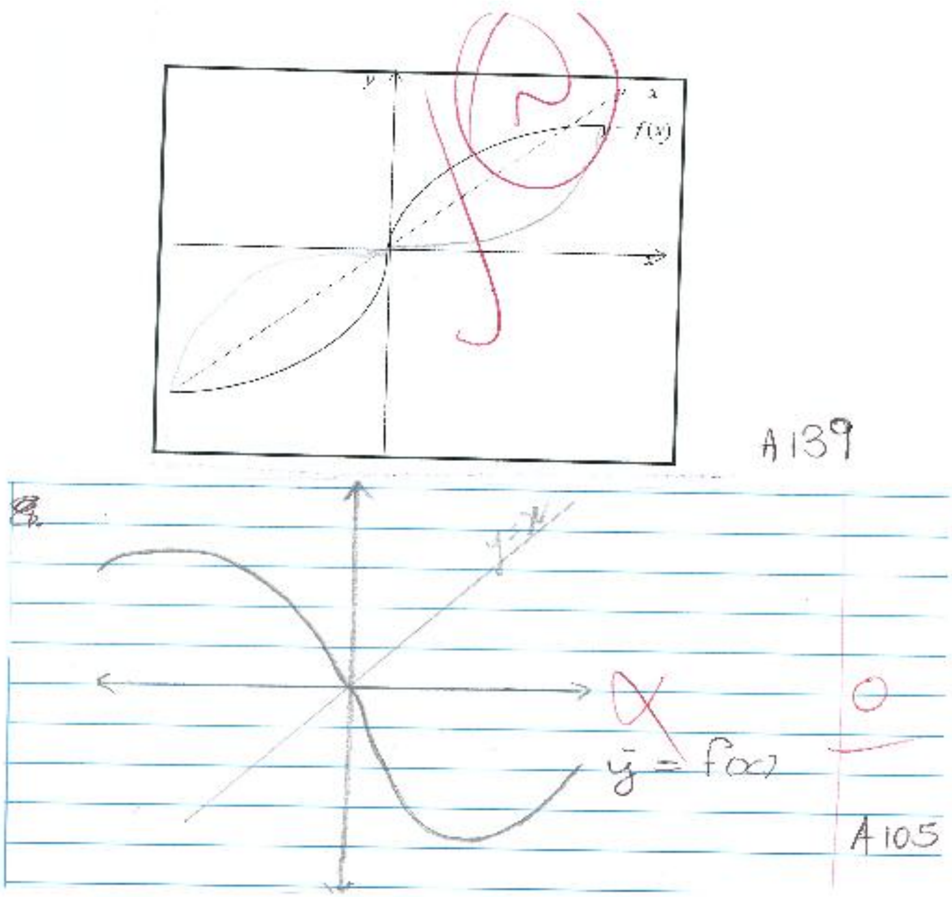


Figure 5. 6: Comparison group students' imposed inverses of function in Figure 5.6

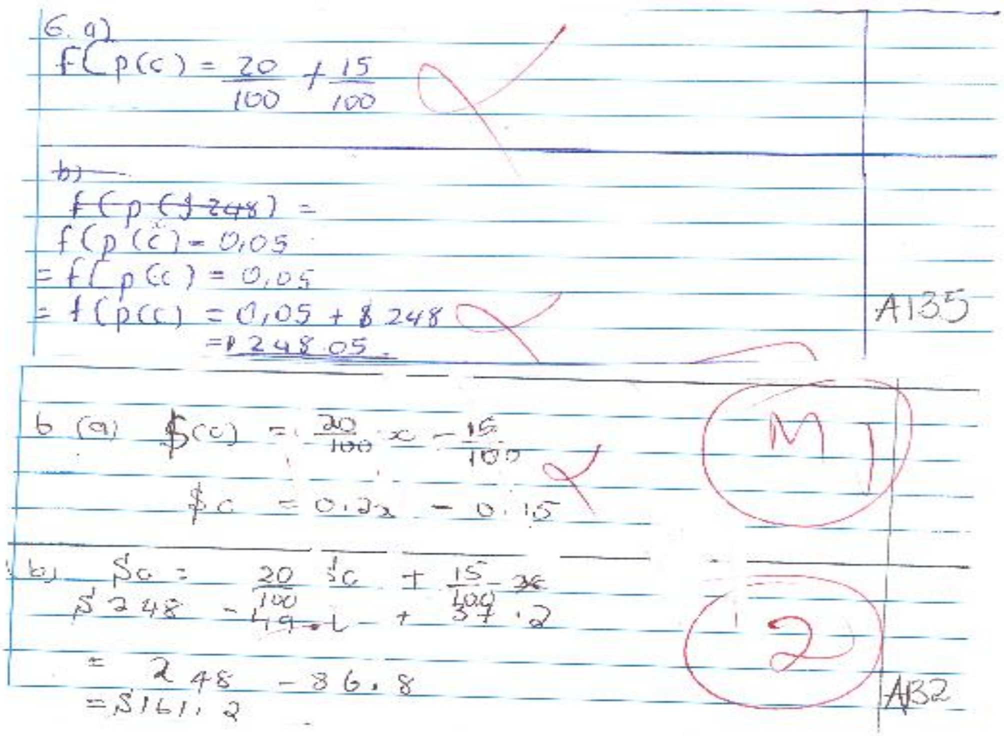
-No idea -A138

5.3.5. Connection of real life scenarios on inverses and compositions of functions

With regard to inverses and compositions of functions, students were given a real life scenario as follows:

During a sale a music drum kit's price was reduced by 20% and preferred customers also received a further 15% discount.

In this question students were asked to (a) formulate a composition function that would represent the final cost price (C) for a preferred customer on a kit that was originally $*c* and also to (b) determine the selling price of a drum kit with a price tag of \$248 when sold to a preferred customer.

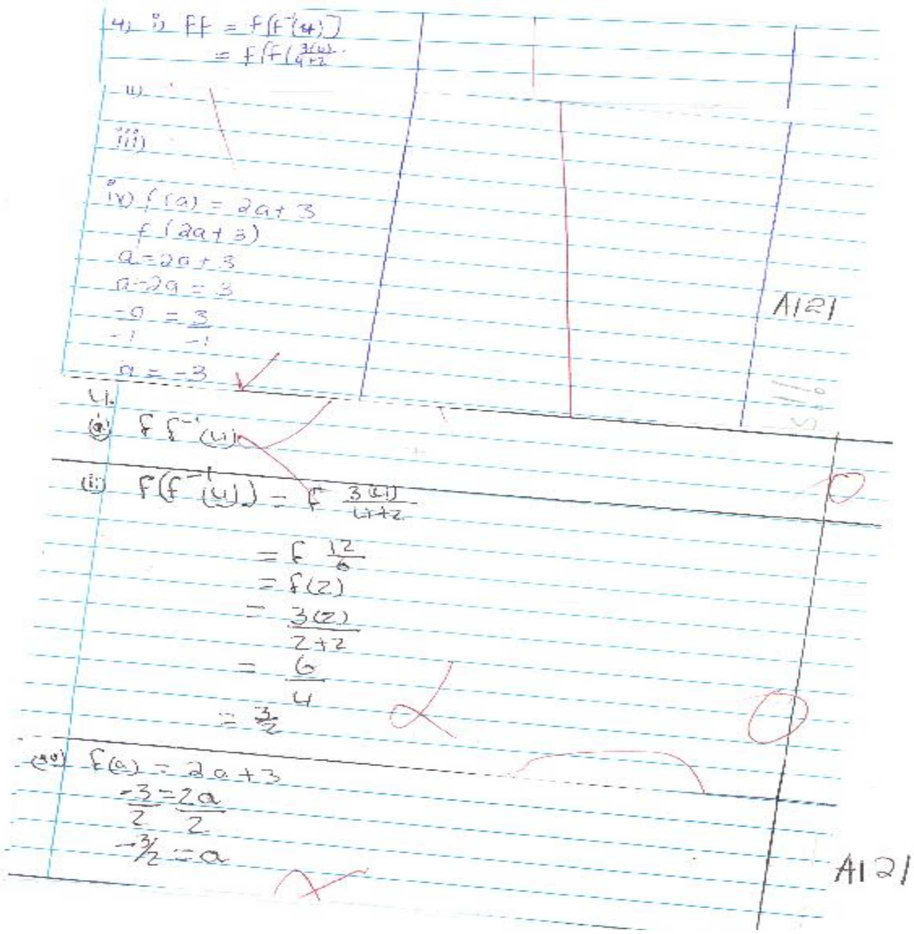

6. a)
$f(p(c) = \frac{20}{100} + \frac{15}{100}$
b)
~~f(p(\$248)~~ =
f(p(c) = 0,05
= f(p(c) = 0,05
= f(p(c) = 0,05 + \$248
= \$248.05
A135

6 (a) $\$(c) = \frac{20}{100}x - \frac{15}{100}$
\$c = 0.2x − 0.15
M1
b) $\$c = \frac{20}{100}\$c + \frac{15}{100}$ − ~~36~~
\$248 − 49.6 + 37.2
= 248 − 86.8
= \$161.2
2
AB2

Figure 5. 7: Comparison group students' responses on connection of real life scenarios on inverses and compositions of functions

5.3.6. Manipulation of abstract mathematical problems involving inverses and compositions of functions (D5)

The pre-test also assessed the students' ability to find the inverses and compositions of functions. The students were given two inverses and compositions of functions: $f: x \rightarrow 3x+4;\quad g(x): x \rightarrow \frac{x}{2}$ and were required to evaluate $a)\ ff^{-1}(-3)\quad b)$ Obtain the inverse of the composite function *fg(x)* c) showing that $g^{-1}f^{-1} = (fg)^{-1}$ and d) solving for *a* given that *f (a) =3*.



4) i) $ff = f(f^{-1}(4))$

$= f(f(\frac{3(4)}{4+2}$

ii)

iii)

iv) $f(a) = 2a + 3$

$f(2a+3)$

$a = 2a + 3$

$a - 2a = 3$

$\frac{-a}{-1} = \frac{3}{-1}$

$a = -3$

A121

4.

(i) $f f^{-1}(4)$

(ii) $f(f^{-1}(4)) = f \frac{3(4)}{4+2}$

$= f \frac{12}{6}$

$= f(2)$

$= \frac{3(2)}{2+2}$

$= \frac{6}{4}$

$= \frac{3}{2}$

(iv) $f(a) = 2a + 3$

$\frac{-3}{2} = \frac{2a}{2}$

$-\frac{3}{2} = a$

A121

Figure 5. 8: Comparison group's manipulation of abstract mathematical problems involving inverses and compositions of functions

5.4. EXPERIMENTAL GROUP PRE-TEST RESULTS ANALYSIS

The pre-test results for the experimental group were also analysed in terms of the same domains as in the comparison group, namely: understanding of the definition of a function (D1), representation of a function (D2), ability to connect compositions and inverses of functions to the real world (D3), ability to formulate the compositions and inverses of functions (D4), manipulation of abstract mathematical problems involving compositions and inverses of functions (D5).

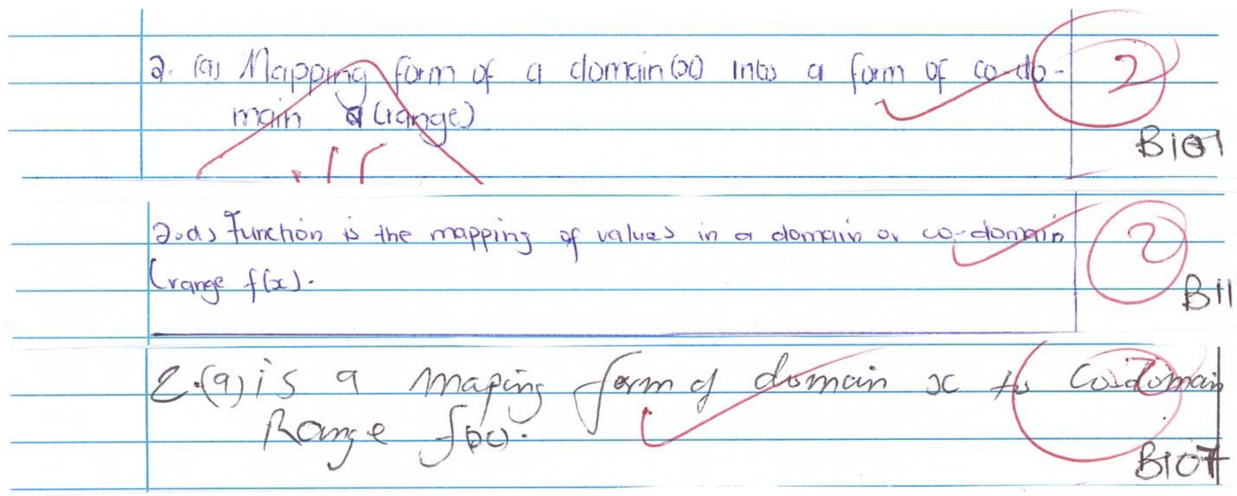
Table 5.4: Experimental group's performance by HLT domain in the pre-test

Domain	**Hypothetical Learning Trajectory Domain**	**Domain Total Marks**	**% weight**	**total marks obtained by 40 students**	**Total possible marks by 40 students**	**pre-test % pass for experimental group**
D1	Understanding of the definition of a function.	7	10.0	116	280	41.4
D2	Representation of a function.	10	14.3	209	400	52.3
D3	Ability to connect inverses and compositions of functions to the real world.	28	40.0	518	1120	46.3
D4	Ability to formulate the inverses and compositions of functions.	14	20.0	229	560	40.9
D5	Manipulation of abstract mathematical problems involving inverses and compositions of functions.	11	15.7	231	440	52.5
	Total	**70**	**100**	**1303**	**2800**	**46.5**

Table 5.4 shows that domain D1 which had 10 % of the test items had a pass rate of 41.5, D2 which had 14.3% of the items had a pass rate of 52.0%, D3 which had 40% of the items had a pass rate of 46.3%, D4 domain which had 20.0% of the items had a pass rate of 40.9% and finally domain D5 which had 15.7% of the items had a pass rate of 52.5%. An overall pass rate of 46.5% was recorded for the experimental group at pre-test.

5.4.1 Definition of a function (D1)

Most students in this group gave different definitions of inverses and compositions of functions as follows.



2. (a) Mapping form of a domain (x) into a form of co-do-main (range) 2 B101

2.a) Function is the mapping of values in a domain or co-domain (range f(x). 2 B11?

2.(a) is a maping form of domain x to codomain Range f(x). B107

Figure 5. 9: Experimental group's definition of a function

-Is the term used to describe something to get an actual meaning of it-B140

5.4.2. Representation of a function (D2)

Students in the experimental group were also required to show using examples their understanding of a function and its representation and were asked to use any relevant ways they knew to represent inverses and compositions of functions. With regard to the representation of a function, students in the comparison group gave the following answers: B109 tried to represent the function *f(x) =1+x*

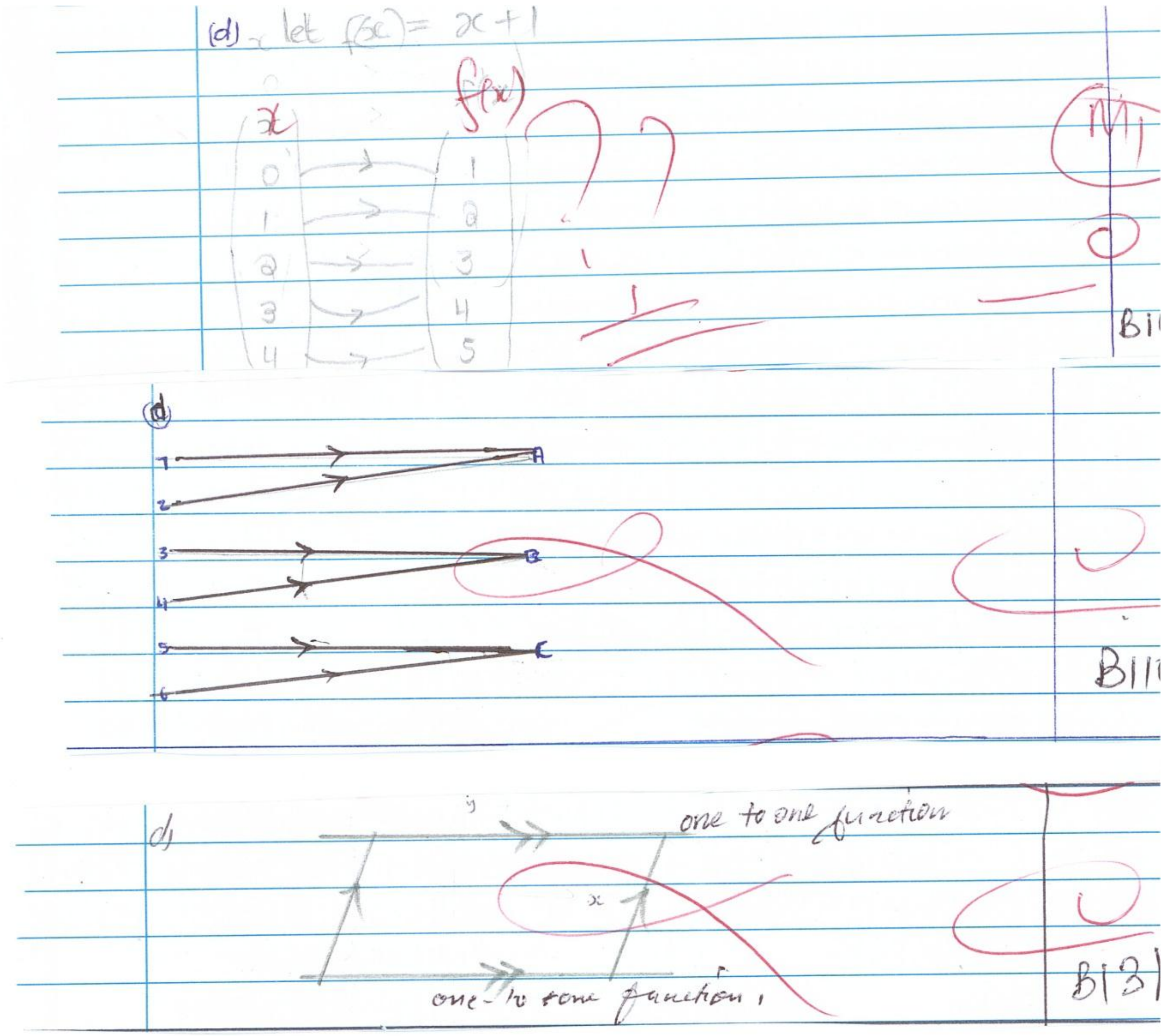


Figure 5.10: Experimental group's representation of a function

5.4.3. Connection of inverses and compositions of functions to the real world (D3)

In this category students were assessed on their ability to come up with relevant examples of inverses and compositions of functions in the real world and using the concept of inverses and compositions of functions to real life problems on given scenarios. One of the scenarios the students had to work on was as given above already (see 5.3.3).

-No idea –B128

-No idea B138

- a) $C=3.30+2.40(k)

b) No idea -B135

-I don't Know-B143

5.4.4. Formulation of inverses and compositions of functions (D4)

The experimental students were also asked to formulate and graphically interpret compositions and inverses of functions and to come up with relevant examples of inverses and compositions of functions. Their responses were as follows:

-No idea -B128

-see below B112; B106; B127

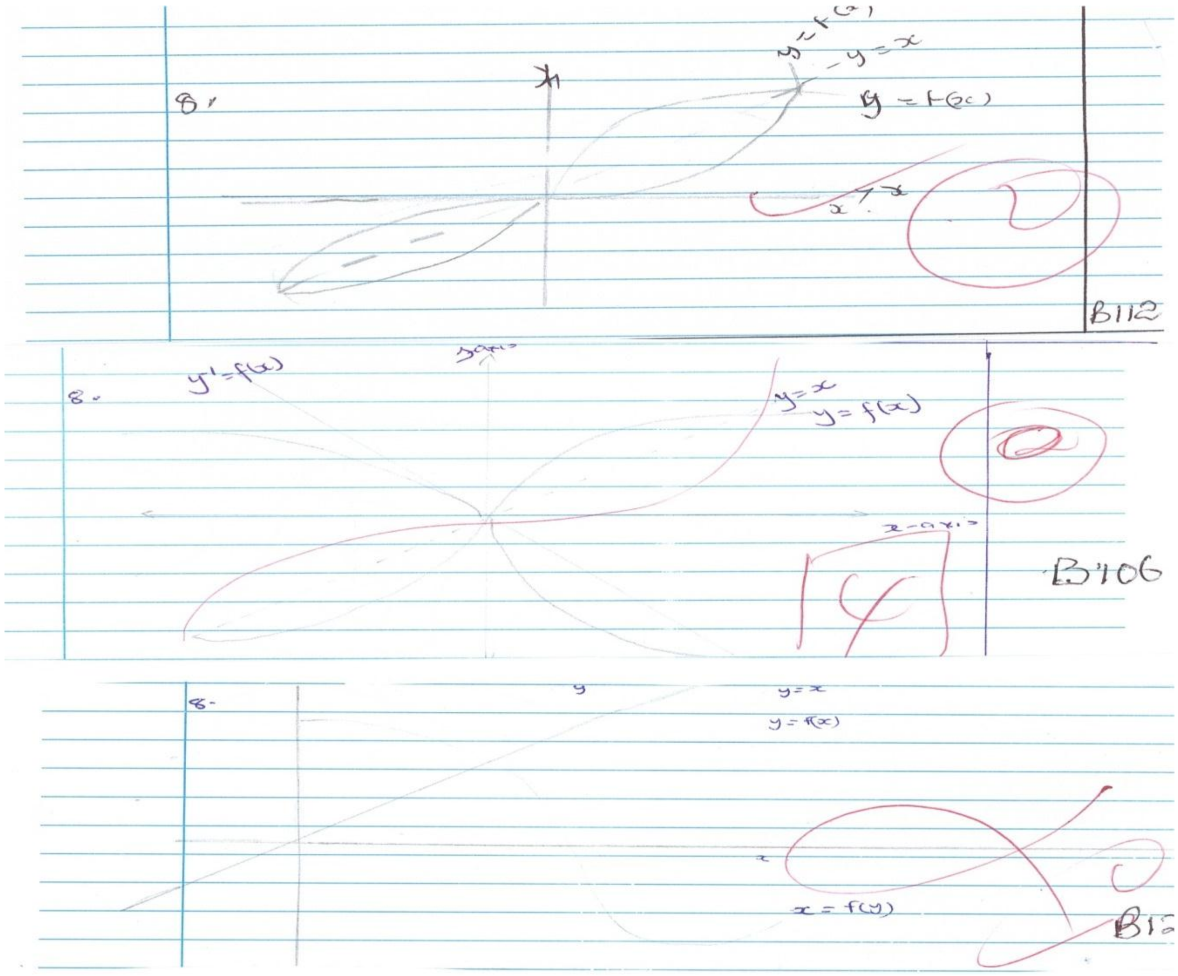


Figure 5.11: Experimental group's responses on the formulation of inverses and compositions of functions

With regard to inverses and compositions of functions, students were given a real life scenario as for the comparison group.

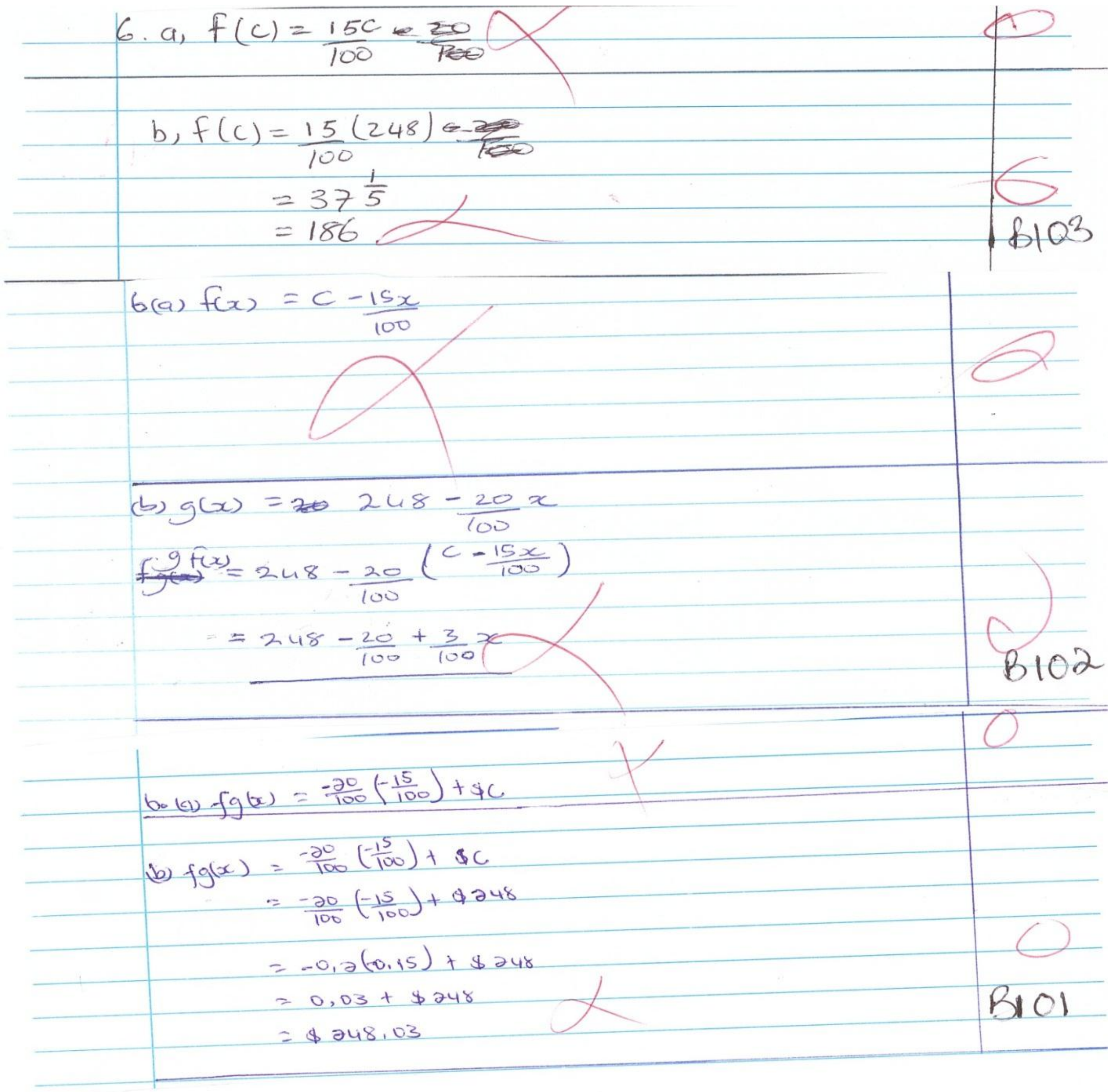


Figure 5.12: Experimental group's responses on real-life applications of inverses and compositions of functions.

5.4.5 Manipulation of abstract mathematical problems involving inverses and compositions of functions (D5)

As for the comparison group the pre-test also assessed the students' ability to find the inverse of a function and finding inverses and compositions of functions and their sample responses were as follows:

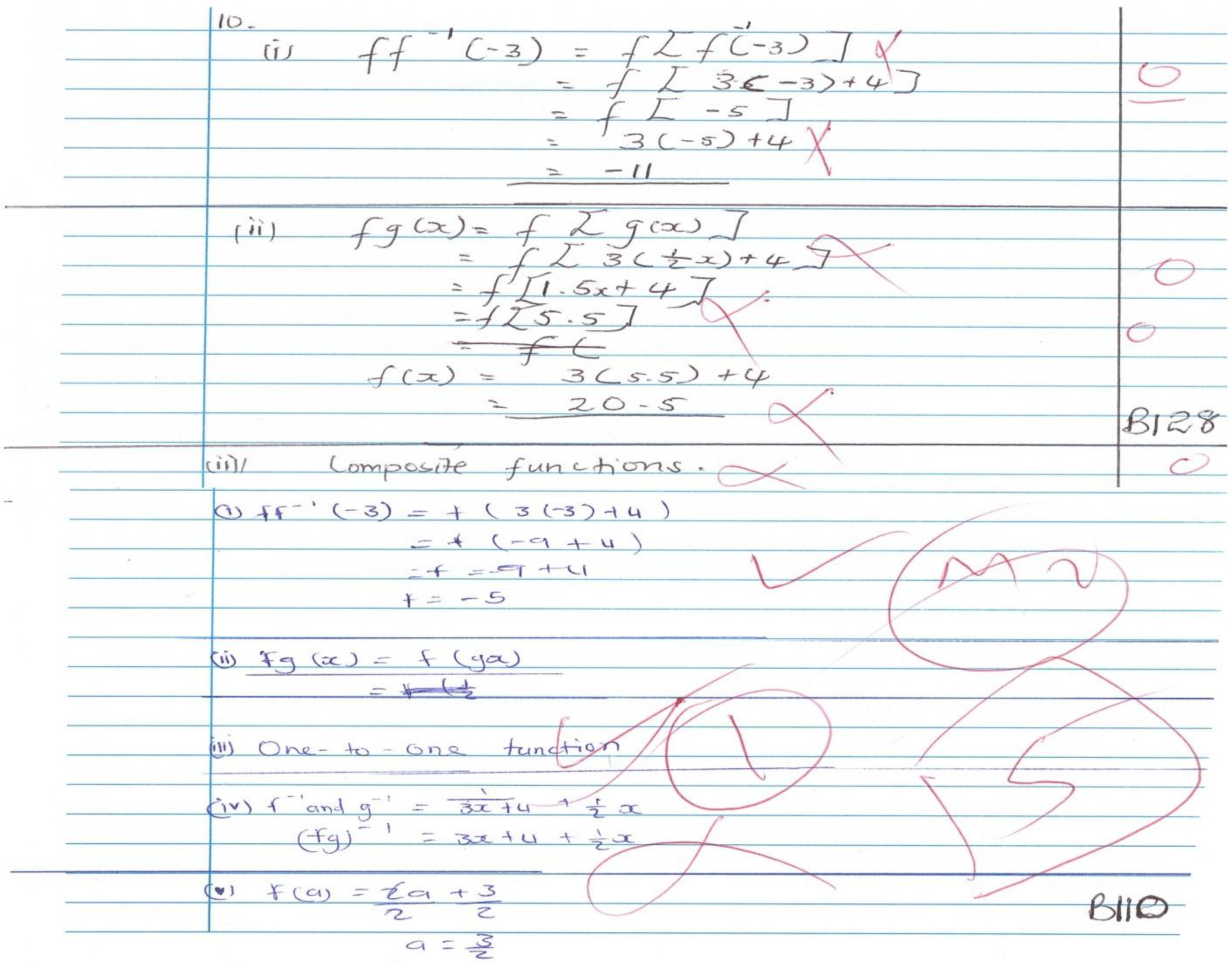


Figure 5.13: Experimental group's responses on manipulations of abstract mathematical problems involving inverses and compositions of functions

5.5. COMPARATIVE ANALYSIS OF THE PRE-TEST RESULTS FOR THE COMPARISON AND THE EXPERIMENTAL GROUPS

Students in the two groups had an idea of what a function is from a mathematical perspective depending on their different mathematical backgrounds from high school.

Table 5. 5: Comparison of experimental and comparison group's performance by HLT domain at pre-test

HLT Domain	Hypothetical Learning Trajectory Domain	Domain Total Marks	% weight	pre-test % pass for X_1group	VarianceX1 Group	pre-test % pass for X_2group	Variance X2 Group	z-test result for independent groups	Critical value of z for at α=0.05	Decision on null hypothesis
D1	Understanding of the definition of a function.	7	10.0	41.5	149.1	43.2	57.1	-0.329	1.96	Accept H_0
D2	Representation of a function.	10	14.3	52.3	130.5	49.5	67.8	1.257	1.96	Accept H_0
D3	Ability to connect inverses and compositions of functions to the real world.	28	40.0	46.3	98.9	47.1	49.6	-0.415	1.96	Accept H_0
D4	Ability to formulate the inverses and compositions of functions.	14	20.0	40.9	114.5	23.0	76.6	8.189	1.96	Reject H_0
D5	Manipulation of abstract mathematical problems involving inverses and compositions of functions.	11	15.7	52.5	154.6	50.2	74.9	1.113	1.96	Accept H_0
	Total	**70**	**100**	**46.5**		**42.8**				

Table 5.5 shows that the pre-test the acceptance of the null hypothesis for D1, D2, D3 and D5 shows that there were no significant differences in the performances of the students in the two groups. However, for domain D4 there was a significant difference in the performances of the two groups and was in favour of the experimental group. This was probably caused by a higher variance on the experimental group as compared to

the variance of the comparison group. These results further suggest that the academic backgrounds for the foundation students were almost the same before the implementation of the treatment.

The Namibian mathematics curriculum prescribes the teaching of the function concept from grade four. However, the inverses and compositions of functions as a topic are emphasized more in the higher level mathematics for the Namibian curriculum and not in the core or extended syllabi. Therefore those students from both groups who could define a function (D1) precisely made use of their knowledge from high school. However, there were some students who had no idea of what a function was and could not even make an attempt to define it. Some tried to define it but their definitions were far detached from mathematical inverses and compositions of functions. Between the two groups the property of a function as a mapping was common. Though some students tried to define a function from the perspectives of domain and co-domain there was an occasional mix up between what the domain and co-domain were.

The definitions given by most students in both groups were in line with Joseph Fourier (1768-1830)'s definition of a function as a relationship between two variables (a dependent) and an independent variable. Students from both groups also tried to define a function from the perspective of a mapping from *x* to *f(x)* and were also able to represent inverses and compositions of functions using graphical mappings.

With regard to the connections of inverses and compositions of functions to real world (D3), some students from both groups were able to demonstrate their understanding of inverses and compositions of functions on the scenarios which were given. However, there were some students who could not get the sense of the question and could not apply their previous knowledge of inverses and compositions of functions to solve problems on the scenarios presented.

The formulation of inverses and composition (D4) was poorly done by both groups. The formulation of the inverse of a one-to-one function requires students' prior knowledge of subject of the formula and solving of equations. This poor performance could be a result of the FP students' inability to manipulate algebraic expressions (D5) which is the cornerstone of equation solving. Kimani (2008) contends that inverses and compositions of functions are central to many branches of mathematics and if a student has a deep understanding of inverses and compositions of functions, he will be able to quickly make mathematical connections that are needed in understanding scientific and other related concepts. Even and Bruckheimer (1998) also argue that students with a deep understanding of inverses and compositions of functions are able to represent them in graphical notation, switch from one form of a function to another, express inverses and compositions of functions verbally and represent them in the form of an equation. At there was no significance differences in the performances of the two groups across all domains as confirmed by the *z* hypothesis test shown in the table above.

5.6. LESSON OBSERVATION RESULTS

The first and fourth lessons for both groups were observed. Findings of the observed lessons are presented below for each group.

5.6.1. Lesson 1 observation protocol results for the comparison group

The lecturer commenced the lesson by greeting the class and cracked a joke to set a congenial classroom environment. Students laughed and it could be seen that they were ready to learn. The lecturer then made a quick recap on the objectives of the lesson for the day while the students (cups to be filled to overflowing) were quiet. The lecturer then wrote the function $f(x) =4x + 1$ and asked the students who were seated in their various groups to read and discuss what was written on the chalkboard. Students discussed for five minutes and then the lecturer asked them to say out what they had agreed was written on the chalkboard. Seven of the group representatives read the function as an equation without mentioning the word function while only one group read it as function. The lecturer then read it correctly as a function.

The lecturer then asked the students what $f(x)$ would be when $x=0, 1,2,3,4$ and the students gave their different values of $f(x)$ for each of the stated value of x. The teacher then wrote the function $f(x) = x^2+4x+5$ on the chalkboard and asked the students to find the corresponding values of $f(x)$ for values of $x=0, 1, 2, 3, 4$. The lecturer then asked the students to discuss in their various groups of four or five a suitable definition of a function. Various definitions of a function were presented by the eight groups:

-A function is a listing of values of x that gives corresponding values of f(x) - GP A.

-A function gives values of f(x) for each substituted value of x-GP D

-A function serves the purpose of merging values of x to those of f(x)-GP H

The researcher used examples of inverses and compositions of functions for the students to derive their own definitions of a function. Finally, the teacher gave the definition of a function to the students and then four more examples of inverses and compositions of functions; a linear, a quadratic, exponential and trigonometric function and asked the students to find their corresponding values of *f(x)* for specified values of *x.*

Students were divided in groups of fives and they were cooperating together to provide solutions to the problems they were given by the teacher. In their various groups, students had a group spokesperson and who would coordinate the group discussion and present their solutions to the class. After the students were given twenty minutes to discuss their solutions they presented a summary of their group findings to the class while the other students would ask them questions where they needed clarity. The definition of a function was then refined based on the various presentations made by the students. The teacher then asked the students to return their group worksheets so that they could be used as the starting points of the next class which was going to be types of inverses and compositions of functions and their graphical representation.

5.6.2 Lesson 1 observation protocol results for the PBL group

The teacher greeted the students whom he found already seated waiting for the commencement of the PBL session. The definition of the PBL was already done in the workshop and the purpose was already outlined but the teacher just made a recap of these aspects again to kick start the lesson. The procedures which were going to be used in the PBL implementation together with the expected outcomes during the implementation processes were recapped. Students were already in their arbitrarily allocated groups of four or five. Groups were already numbered before students' arrival so students just went to sit at their assigned groups. The classroom was set in such a way that the teacher was accessible to all the groups. The researcher brought with him a stapler and extra reference materials for the students to use during the lesson.

The researcher gave a brief lecture on the relationship between the operation of a car that is fuelled with a specific amount of fuel and covers a certain distance. The researcher went on to explain that if there was no fuel in a car it would not cover any distance. Therefore there is a relationship between the amounts of fuel put and the distances covered. As such we can express the distance covered as a function of the amount of fuel used $f(x) = kx$ where x is the amount of fuel used and f if the distance travelled. The teacher then distributed some papers to the various groups where students had to come up with a suitable definition of a function and a practical illustration that shows a clear understanding of what a function is using real life examples. In order for the students to do this, they had to employ the hybrid model combining the Barrow's Seven Jump and the modified shoestring models for problem based learning as explained (see 2.8.2; 2.8.6). Group members had to collaborate together in coming up with a suitable definition of a function and real life example that resembled a function. Students would then present what they had agreed upon while the other group members would stand to defend their group findings. Below are a few selected definitions of the inverses and compositions of functions and the real-life illustrations which some of the groups presented during the lesson.

-A function is a mapping of values of x to corresponding values of f(x)

Example: The tension in a stretched spring varies directly as the extension. Given that a tension of 6 Newton's produces an extension of 15cm, find the tension required to produce an extension of 20cm. -GPB-C

-A function is a relationship of variables which is such that any input of a variable x gives a resultant output variable in f(x).

Example: John sells bananas by the road side. If he sells 10 bananas, he will get 30 dollars if he sells 7 bananas, he will get 21 dollars. How many bananas did he sell if comes home with 600 dollars? GPB- I.

5.6.3. Comparative analysis of the observations made in the two groups.

The lesson commencement for both groups was the same except that in the experimental group the teacher introduced his lesson with a real world story that became the students' baseline to define inverses and compositions of functions and also to enlighten them on what examples of functions they could come up with. During the teaching and learning of inverses and compositions the comparison group was concentrating more on abstract concepts of inverses and compositions of functions rather than the application and relevance of the concept of inverses and compositions of functions in the real world. Both the comparison and experimental groups gave their feedback the same way. However, there was more emphasis on group participation and collaboration in the experimental than in the comparison group. The comparison group only defined a function but the experimental group went on to give a real world example that resembled a function, and this broadened the experimental group members' understanding of a function. These findings further suggest that the experimental group members may stand greater chances of understanding practical and real-world problems involving calculus (differentiation and integration).

These findings concur with findings of Nguyen (2009) who found out that the conventional teaching approaches which have been in use for decades in higher education have been found to be limiting students in becoming proper problem solvers and self-directed learners in the information age. There the FP experimental group went an extra mile to demonstrate their understanding of compositions and inverses of functions through creating proper real world connections.

5.7. RESULTS OF TUTORIAL OBSERVATIONS

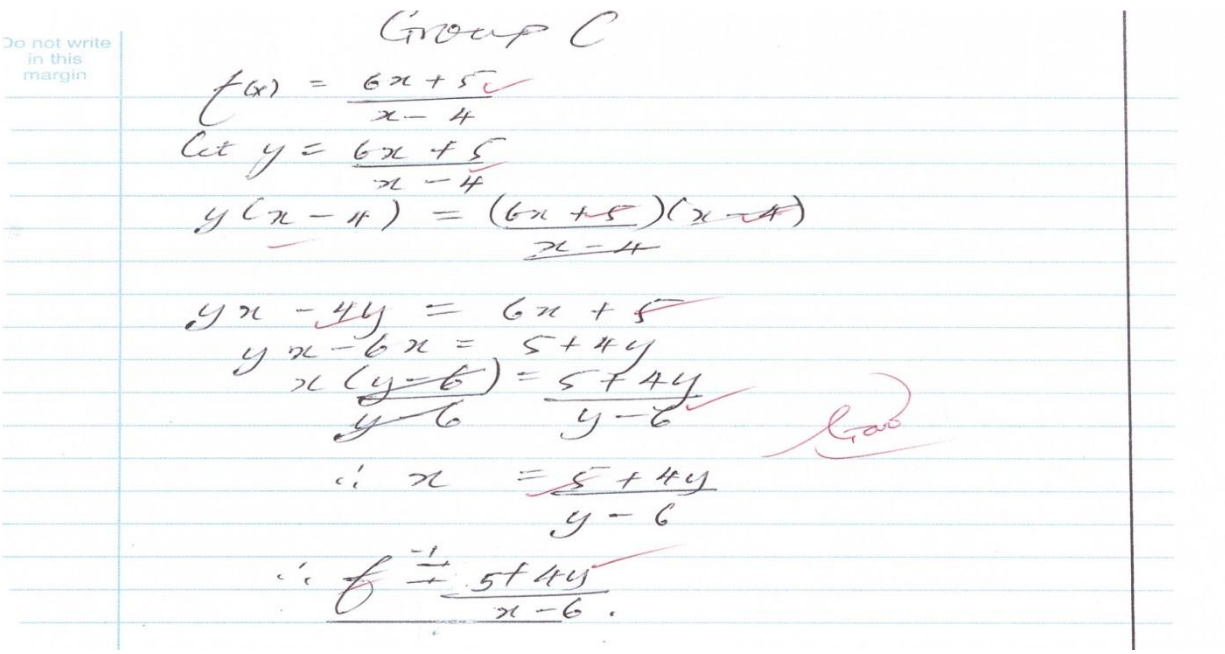
Out of the ten tutorial sessions two tutorials were observed for each of the groups. But there were no distinct differences in the recorded observation protocols made for the two groups. The researcher chose to get proceedings for tutorial 2 being recorded.

During this tutorial session students were working on inverses of inverses and compositions of functions and their graphical representation.

5.7.1. Tutorial 1 observation protocol results for the comparison group

During this tutorial session the students were working on composing inverses of one-to-one functions and representing them graphically. Each of the comparison group students was given a function different from any other group whose inverse had to be found and be presented to the class (see Appendix I).

Group GP-C was given the $f(x)=\dfrac{6x+5}{x-4}$ expression to find its invers


Do not write in this margin

Group C

$f(x) = \dfrac{6x+5}{x-4}$

Let $y = \dfrac{6x+5}{x-4}$

$y(x-4) = \dfrac{(6x+5)(x-4)}{x-4}$

$yx - 4y = 6x+5$

$yx - 6x = 5+4y$

$\dfrac{x(y-6)}{y-6} = \dfrac{5+4y}{y-6}$

Good

$\therefore x = \dfrac{5+4y}{y-6}$

$\therefore f^{-1} = \dfrac{5+4y}{x-6}.$

Figure 5.14: Comparison group's results on the observed item

-With regard to the presentation and interpretation of the inverse they could not perform in since they did not know how to draw the graph of f(x).

-GP-C

After the presentation by GP-C another group GP-F also made their presentation of the inverse function and the graphical representation for the function f(x) =x+4.

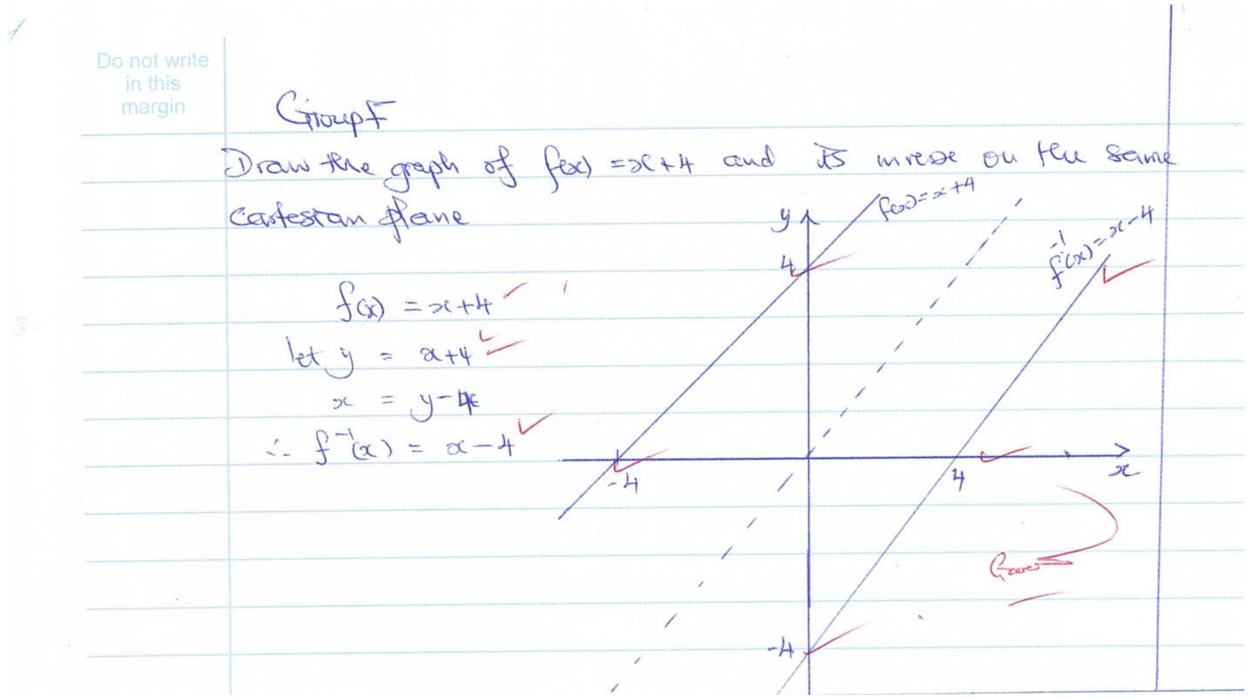


Figure 5.15: Comparison group's observed item for the tutorial

-GP-F

5.7.2. Tutorial 2 observation protocol results for the experimental group

During this tutorial session the experimental group students were working on composing inverses of one-to-one and inverses of functions and were also representing them graphically. Each of the 8 experimental groups were given the responsibility of composing a function in word problems then find and interpret the meaning of the inverse of the function then represent both the function and the inverse on a graph using the definition of an inverse. The results of the observation were shown below.

-*The cost of printing wedding invitation cards is partly constant and partly varies as the number of cards printed. If the cost of 50cards is $24 and the cost of printing 100 cards is $46.50.*

Formulate a function of C in terms of n.

Find the cost of printing 80 cards.

Write down and interpret the inverse of the function C (n).

Represent the function C (n) and its inverse on the same graph using the definition of an inverse of a function.

$$
\begin{aligned}
&i) Let\ the\ of\ \ pr\mathrm{int}\,ing\ n\ cards\ be\ \$C.\\
&Then\ C = h + kn\\
&substitute, n = 50, \quad C = 24\\
&\Longrightarrow 24 = h + 50k..............................(i)\\
&substitute, n = 100, C = 46.5\\
&\Longrightarrow 46.5 = h + 100k..........................(ii)\\
&(ii) - (i) \quad 22.5 = 50k\\
&\Longrightarrow k = 0.45\\
&substitute\ in\ \ (ii), 46.5 = h + 45
\end{aligned}
$$

$$
\begin{aligned}
&\therefore h = 1.5.\\
&Hence\ C = 1.5 + 0.45n
\end{aligned}
$$

$$
\begin{aligned}
&ii) The \quad \cos t \quad pr\mathrm{int}\,ing \quad 80 \quad cards\ \ is\ C = 1.5 + 0.45(80)\\
&\qquad\qquad\qquad\qquad\qquad\qquad C = \$37.50
\end{aligned}
$$

$$
\begin{aligned}
&iii) \quad C(n) = 1.5 + 0.45n\\
&let\ \ y = 1.5 + 0.45n\\
&\Longrightarrow y - 1.5 = 0.45n\\
&\therefore n = \frac{y - 1.5}{0.45}
\end{aligned}
$$

$$C^{-1}(n) :\longrightarrow \frac{n - 1.5}{0.45}$$

***Interpretation**: The inverse of C is and expression which can be used to find the number of cards which are worth a particular amount of money*

iv) $C^{-1} : n \to \dfrac{n - 1.5}{0.45}$

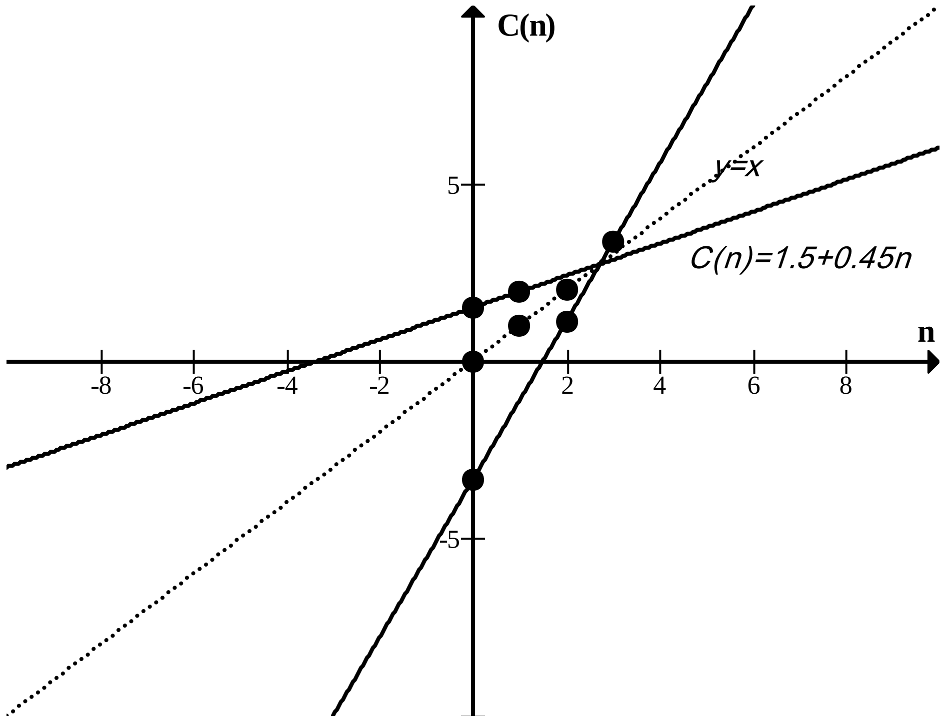


Figure 5.16: Experimental group's observed tutorial item

- GPB-A

The population (p) of a city increased from 23 400 to 27 800 between 2002 and 2006.

a) *Write down an expression for the rate at which the population is increasing*
b) *What will be the population size in the year 2013?-GPB-D*

You are an 18 year old high school student. You need to decide whether you will continue with your education or beginning working. No matter what you decide, you must pay for your expenses (variable) (food, cell phone, car, bills, power etc.).The benefit of working after school is that you begging earning money immediately. If you continue with your education, you will go to the college and pay for your own education along with all the other variables you may need to make things work for you.

-Realising that life is a composition function made of many other functions/ variable. The decision which an individual makes in life determines the level of success one can make

in life after considering all the other inverse variables associated with not following a particular protocol for a successful life. It is therefore important for one to be able to rule out what works to your disadvantage for you to be a successful person in life.

Group F

5.7.3 Comparative analysis of the comparison and the experimental group observation protocol results for Tutorial 2 observation

Students from the two groups were working on composing inverses and compositions of functions and their inverses and presenting them on a graph using the property of an inverse that it is a reflection of the graph of *f(x)* along the line *y=x.* Both groups could find the inverse of a function. However, in the case of the comparison group the focus was on finding the inverse of the function. For the experimental group the focus was on showing that one understands the meaning of a function and its inverse and being able to interpret the relationship between the two, and demonstrating their understanding by representing the function together with its inverse on a graph. The comparison and experimental groups could find and represent the inverse of a function graphically but the aspect of interpretation of the meaning of what was represented was better understood and demonstrated by the experimental group. Therefore these findings corroborate those of Mergel (2008) who noted that PBL implementation enhances student self-direction and autonomy. However, Mergel advises that some adjustments must be made in the programme to improve all the other necessary skills so that the PBL product is holistically skilled.

A closer look shows that the objectives and purposes of PBL as a philosophy of learning are different from those of the mathematics for science foundation which simply deals with abstract understanding of inverses and compositions of functions and without necessarily applying them in the real world context. On this note Dahlgren (2004) concludes that the evaluation of PBL implementation is such a complex phenomenon because the objectives may be focused on process or content whereas the outcomes may be relative to the pre-set objectives of a programme.

5.8. ANALYSIS OF THE POST-TEST RESULTS FOR THE COMPARISON AND THE EXPERIMENTAL GROUPS

After the two groups went through the processes of teaching, observations and tutorial sessions in order to reach the conjectured HLT, they were both subjected to a post test. The purpose of the post-test was to find out whether there was any change in terms of the performances of the two groups and also to see if the observed change could be attributed to PBL, chance or other attributes. The pre-post-tests comprised of questions based on students understanding of the inverses and compositions of functions, representation of inverses and compositions of functions, the nature of a one-to-one inverses and compositions of functions and the conditions necessary for a function to be invertible, evaluation of a function, understanding the definition of the nature of the inverse of a function from a graphical perspective, problems on real world applications of inverses and compositions of functions and their inverses and solving equations involving inverses and compositions of inverses and compositions of functions. Table 5.6 below shows a summary of results of the two groups.

Table 5.6: Summary for the post-test results for both groups (N=40)

Group	Range	Minimum	Maximum	Percentage Mean Score		Std. Deviation	Variance
					Std. Error		
Comparison	58	20	78	72.4	1.262	11.4	129.96
Experimental	40	42	82	79.5	0.994	8.97	80.58

Table 5.6 above shows that the comparison group had a percentage mean score of 72.0 with a standard deviation of 11.4, while the experimental group had a percentage mean score of 79.5% with a standard deviation of 8.97. It can be seen that the experimental group had a higher mean than the comparison group. The mean difference between the two groups was 7.1 percentage points in favour of the

experimental group. At a glance these results suggest that the experimental group performed significantly higher than the comparison group. The researcher performed a two tailed hypothesis test for the differences of the means of the two groups to find out if there was a significant difference in the performances. The results of the two tailed *z* test showed that ($Z_{\mathbf{Calculated}}$=10.64; $Z_{\mathbf{Standard}}$=1.96; α=0.05) and rejected the null hypothesis which stated that (H_0:$\mu_E=\mu_C$) in favour of the alternative which stated that (H_1:$\mu_E-\mu_C \neq 0$).These results show that after the implementation of the treatment, the performance of the experimental group was higher significantly better than that of the comparison group.

5.8.1. Comparative analysis of the post-test pre-test

The post-test results for the two groups were analysed in terms of the following domains: understanding of the definition of a function (D1), representation of a function (D2), ability to connect inverses and compositions of functions to the real world (D3), ability to formulate the inverses and compositions of functions (D4), manipulation of abstract mathematical problems involving inverses and compositions of functions (D5). Table 5.7 presents the comparison group results for each HLT domain.

Table 5.7: Comparison group's pre-post-test results analysis by HLT domain

	Hypothetical Learning Trajectory Domain	**Domain Total Marks**	**% weight**	**pre-test % pass rate for comparison group**	**Variance(Pre)**	**post-test % pass rate for comparison group**	**Variance(Post)**	***z*-test result for dependent groups**	**Critical value of *z* for α=0.05**	**Decision on null hypothesis**
D1	Understanding of the definition of a function.	7	10	43.2	57	78.6	157	6.61	1.96	Reject H_0

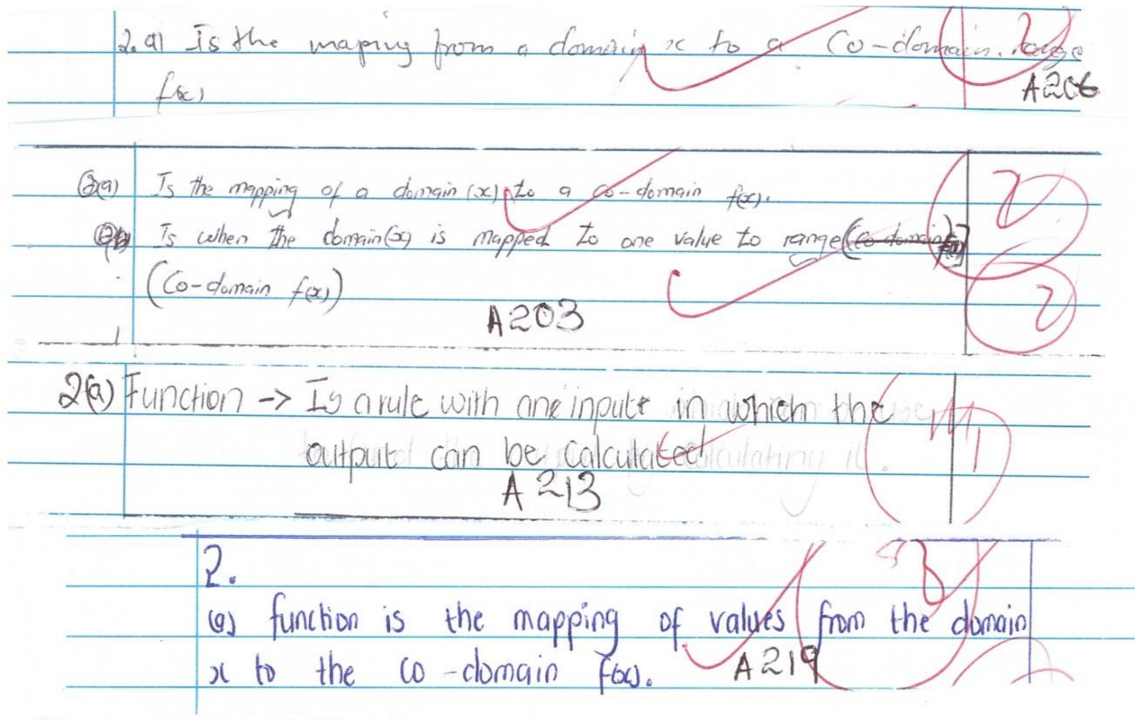
D2	Representation of a function.	10	14	49.5	68	60.3	148	4.65	1.96	Reject H_0
D3	Ability to connect inverses and compositions of functions to the real world.	28	40	47.1	50	77.6	89.6	16.34	1.96	Reject H_0
D4	Ability to formulate the inverses and compositions of functions.	14	20	23.1	77	71.1	107	22.43	1.96	Reject H_0
D5	Manipulation of abstract mathematical problems involving inverses and compositions of functions.	11	16	50.5	75	78.4	144	11.92	1.96	Reject H_0
	Total	**70**	**100**	**40.6**		**72**				

At 95% level of significance it can be concluded that there is a significant difference in the performances of the comparison group by HLT domain D1-D5. These results suggest that the lecture methods were effective in improving students understanding of compositions and inverses of functions. These results further suggest that the role of the lecture method in the teaching and learning of compositions and inverses of functions cannot be underestimated.

5.8.2. Comparison group results

5.8.2.1. Definition of a function (D1)

All the 40 students in this group gave different definitions of inverses and compositions of functions.


2. a) Is the maping from a domain x to a Co-domain, range f(x)
A206

(2a) Is the mapping of a domain (x) to a co-domain f(x).
(2b) Is when the domain(x) is mapped to one value to range (~~co-domain~~ f(x))
(Co-domain f(x))
A203

2(a) Function -> Is a rule with one input in which the output can be calculated
A213

2.
(a) function is the mapping of values from the domain x to the co-domain f(x).
A219

Figure 5.17: Comparison group's responses on the definition of a function

-Is a relation between two variables x and y such that each x value (independent variable) there is one and only one y value (dependent variable)-A217

5.8.2.2. Representation of a function (D2)

In their experiences of inverses and compositions of functions, students were required to show using relevant examples, their understanding of a function and its representation and were asked to use any different ways they know which can be used to represent inverses and compositions of functions. With regard to the representation of a function, students in the comparison group gave the following answers:

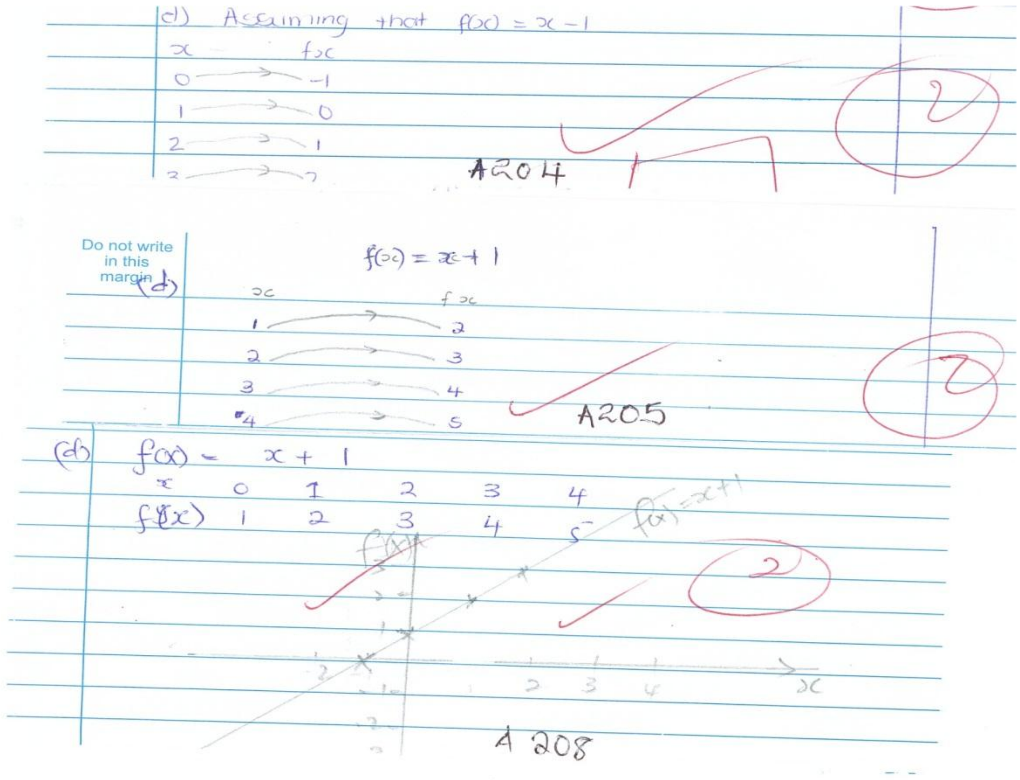


Figure 5. 18: Comparison group's responses on the representation of a function

-No Idea -A218

5.8.2.3. Connection of inverses and compositions of functions to the real world (D3)

In this category students were assessed on their ability to come up with relevant examples that resemble inverses and compositions of functions in the real world and using the concept of inverses and compositions of functions to real life problems on given scenarios.

One of the scenarios which students had to assess was presented as follows: *John is blind but he can hear and can also speak. He hears you talking about composition inverses and compositions of functions and he asks you what a composition function is. How would you explain to him what a function is from a layman's perspectives using a familiar example which John is aware of***?**

In this scenario, the students had to use their understanding of compositions and inverses of functions and familiar examples to explain to another person. That is the reason why John's brother was meant to be blind so that the students could not use their algebraic understanding of functions but real-life example that the brother also knew. As they did this they showed a better understanding of the concept.

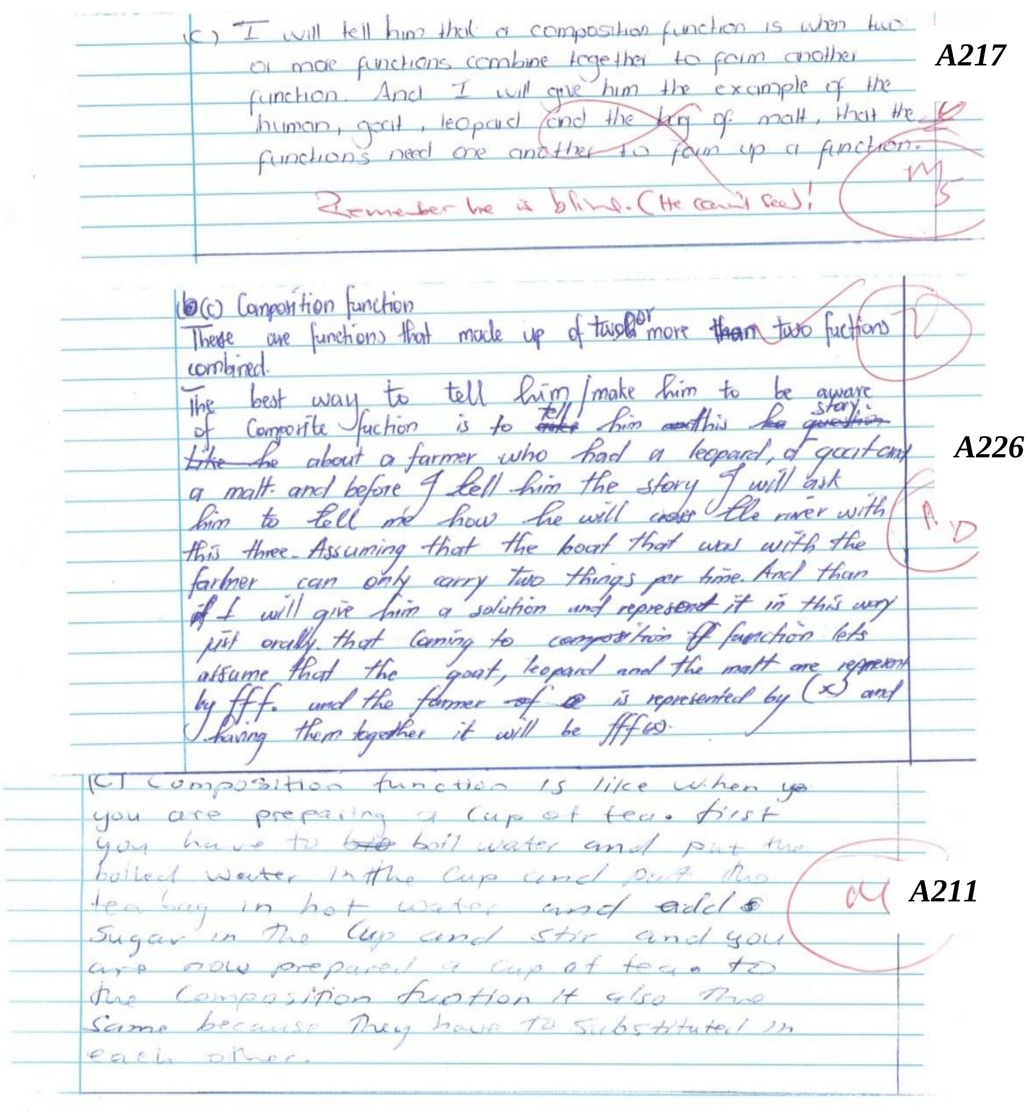


Figure 5.19: Comparison group's responses on the connection of inverses and compositions of functions to the real world

-No idea- A229; A225; A234; A228.

5.8.2.4. Formulation of inverses and compositions of functions (D4)

The students were also asked to formulate and graphically interpret inverses of inverses and compositions of functions and to come up with examples of inverses and compositions of functions. They were given a graph of a function *y=f(x)* and asked to draw the graph of the inverse of the graph of *y=f(x)* as in the pre-test. Their responses were as follows:

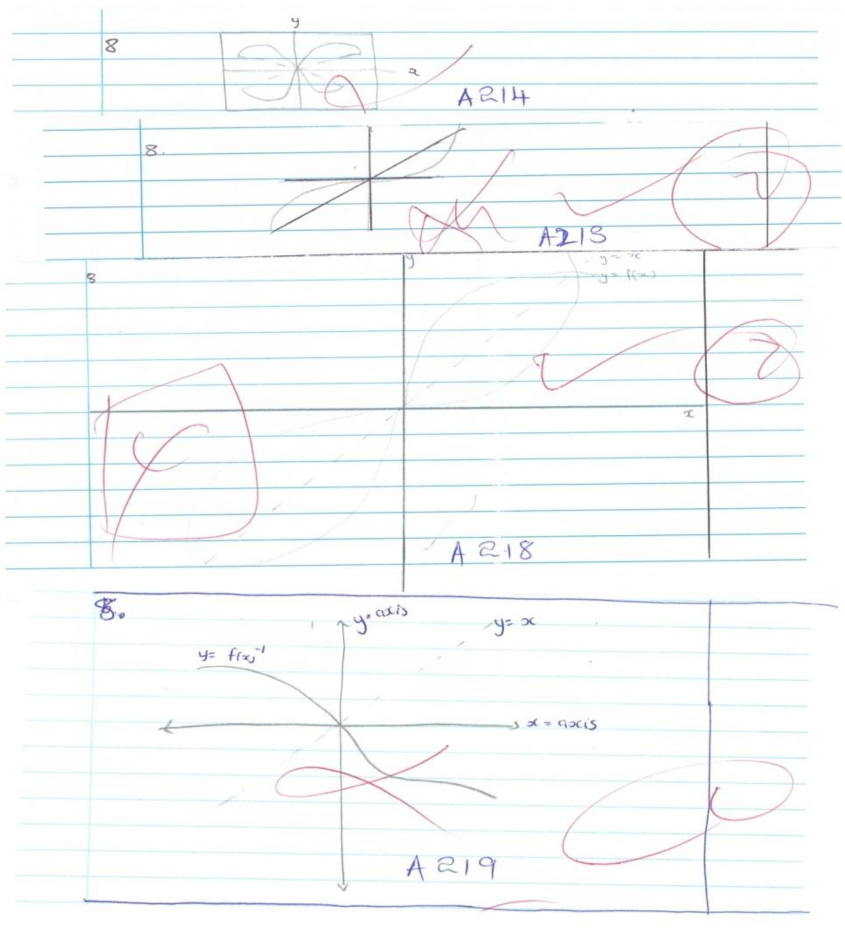


Figure 5.20: Comparison group's responses on the formulation of inverses and compositions of functions

-No idea –A213

With regard to composition inverses and compositions of functions, students were given a real life scenario as in the pre-test (see section 5.8.1.): Their responses were as follows:

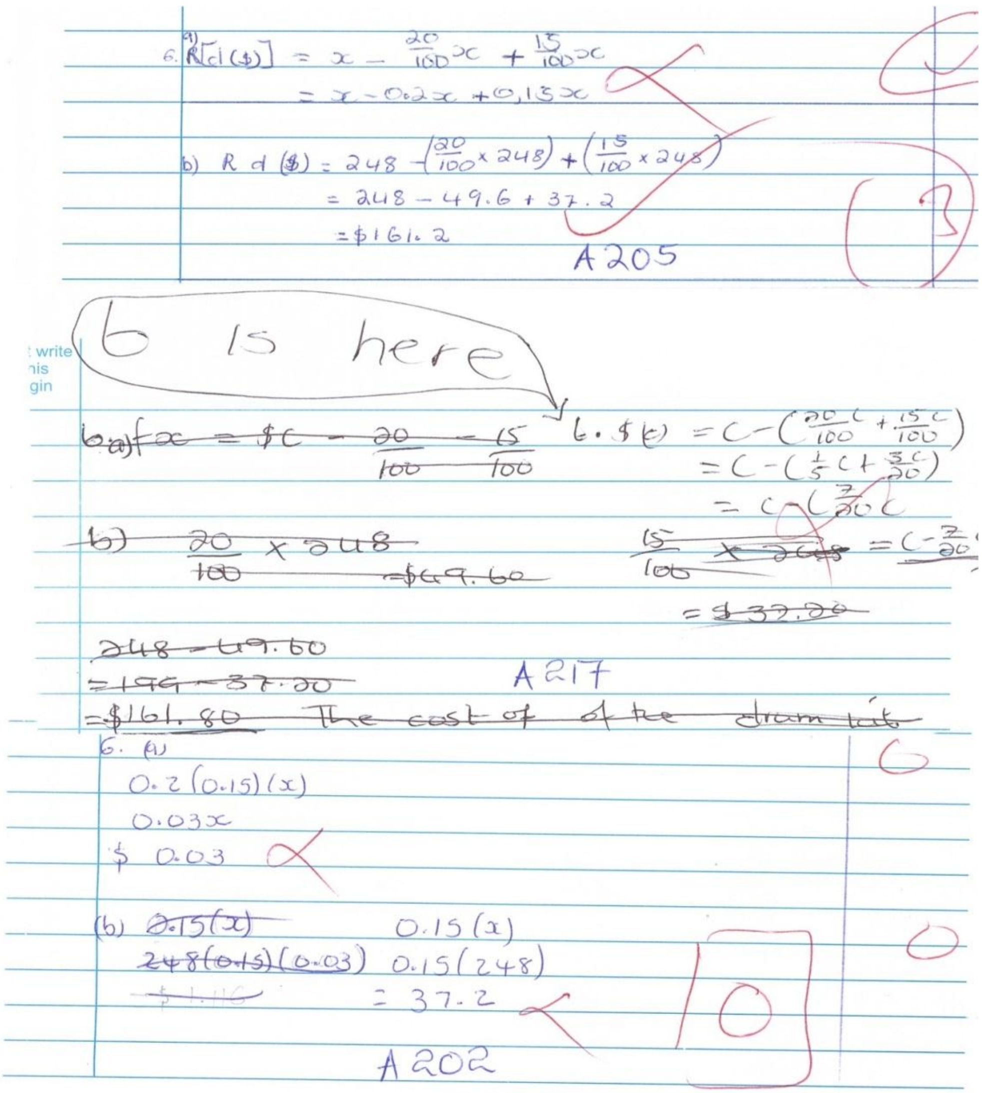


Figure 5.21: Comparison group's responses on real life scenarios involving inverses and compositions of functions.

5.8.2.5. Manipulation of abstract mathematical problems involving inverses and compositions of functions (D5)

The post-test also assessed the students' ability to find the inverse of a function and finding inverses and compositions of functions. The students were given two inverses and compositions of functions as in the pre-test (see section 5.2). Their sample responses were as follows:

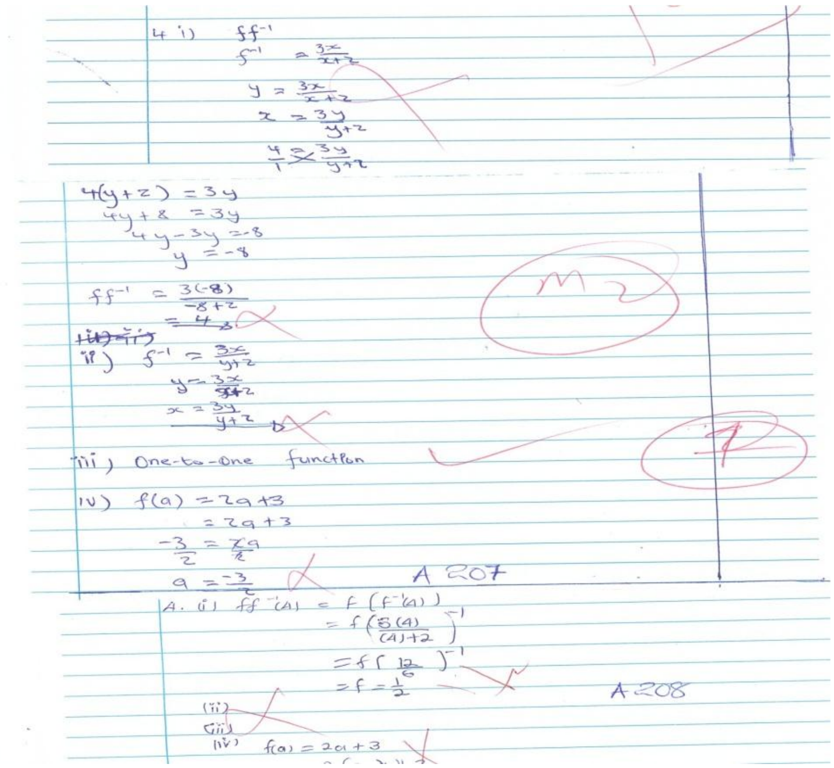


Figure 5.22: Comparison group's responses on the manipulation of abstract mathematical problems involving inverses and compositions of functions

5.8.3. Experimental group

The post-test results for the experimental group were also analysed in terms of the following domains: understanding of the definition of a function, representation of a function, ability to connect inverses and compositions of functions to the real world, ability to formulate the inverses and compositions of functions, manipulation of abstract mathematical problems involving inverses and compositions of functions. Table 5.8 presents the experimental groups performance by each HLT domain.

Table 5.8: Experimental group performance by HLT domain (pre-post)

Domain	**Hypothetical Learning Trajectory Domain**	**Domain Total Marks**	**Domain**	**pre-test % pass for experimental group**	**Variance**	**post-test % pass for experimental group**	***Variance E Group***	***z*-test result for dependent groups**	**Critical value of *z* for α=0.05 and**	**Decision on null hypothesis**
D1	Understanding of the definition of a function.	7	10.0	41.5	149.1	82.9	79.5	17.363	1.96	Reject H_0
D2	Representation of a function.	10	14.3	52.0	130.5	80.3	91.7	12.007	1.96	Reject H_0
D3	Ability to connect inverses and compositions of functions to the real world.	28	40.0	46.3	98.9	82.1	100.4	16.038	1.96	Reject H_0
D4	Ability to formulate the inverses and compositions of functions.	14	20.0	40.9	114.5	71.2	57.9	14.594	1.96	Reject H_0
D5	Manipulation of abstract mathematical problems involving inverses and compositions of functions.	11	15.7	52.5	154.6	80.7	73.0	11.822	1.96	Reject H_0
	Total	**70**	**100**	**46.5**		**79.5**				

The comparative analysis of the pre-test and post test results for the experimental group for domains D1-D5 rejected the null hypothesis in favour of the alternative at 95% level of significance. These results are indicating an improvement of students' marks as a result of the PBL approach. These results do not show us which teaching method was more effective in improving students' performances in the teaching and learning of compositions of functions.

Table 5.8 above shows that experimental group students performed the best in domain D2 with 82.9% and their least performance was in domain D4 with 71.2%. Their overall performance score was 79.4652% and was higher than their pre-test performance score of 46.5%.

5.8.3.1. Definition of a function (D1)

Most students in this group gave different definitions of inverses and compositions of functions as follows:

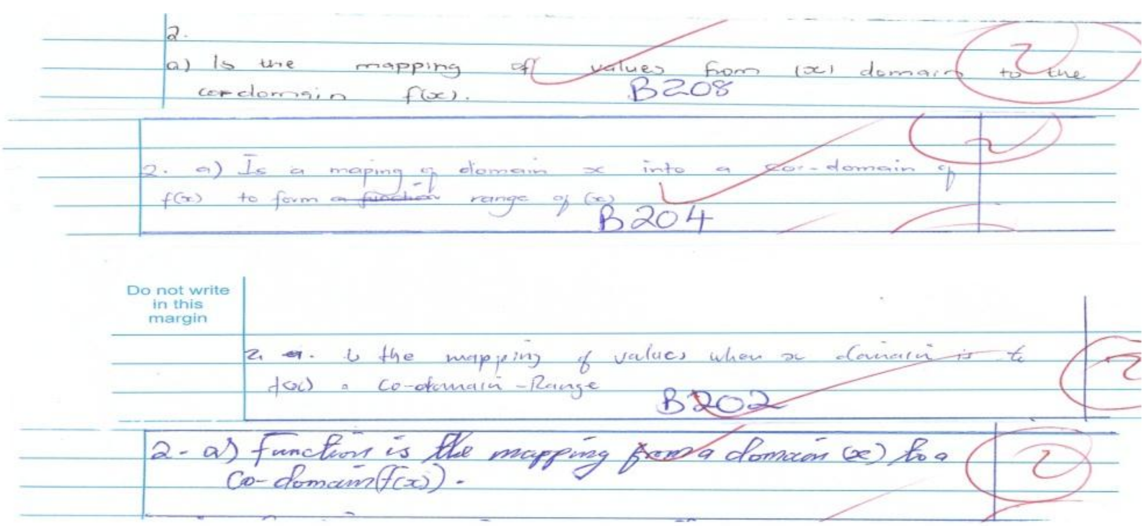


Figure 5.23: Experimental group's responses on the definition of a function

5.8.3.2. Representation of a function (D2)

Students were required to show using examples their understanding of a function and its representation and were asked to use any relevant ways they know which can be used to represent inverses and compositions of functions. With regard to the representation of a function, students in the experimental group gave the following answers:

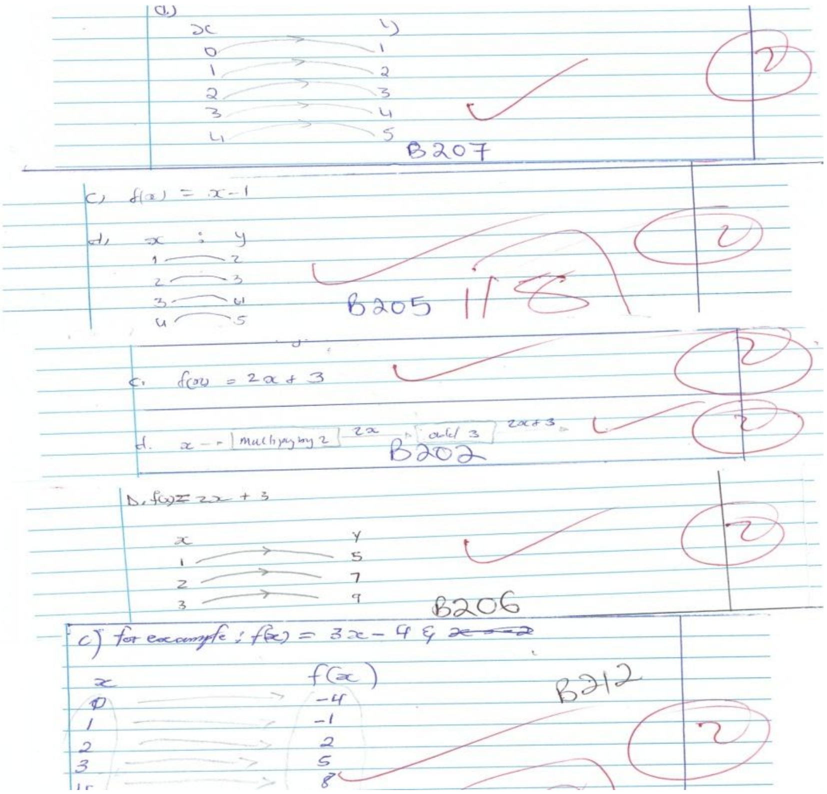


Figure 5.24: Experimental group's responses on the representation of a function

5.8.3.3. Connection of inverses and compositions of functions to the real world (D3)

In this category students were assessed on their ability to come up with relevant examples that resemble inverses and compositions of functions in the real world and using the concept of inverses and compositions of functions to solve real life problems on given scenarios. One of the scenarios the students had to work on was the scenario about a blind man who could hear and speak described earlier above (see section 5.7.2.3).

C)
firstly a composition function is a function of a function. Is made up of 2 or more combination that are also needed or has to be solved out to form only one function that can be used to solve a function.
To solve this function you need to have an equation e.g for fg (x) and for gf(x), so where there is x, you replace that x with the equation of the other composition until you end up having only one function is ~~but~~ each equation
B240

(c) ⇒ Composition is a function of function.
⇒ We can use a function to map another function.
the good example is that, my age is twice than yours.
And my mother's age is 28 times mine.
⇒ Therefore my mom's age 28 times than your age when twices (multiplyd by 2)
⇒ In conclution my mother's age is mapped on to your age.
B201

C. A COMPOSITION ~~FRACTION~~ FUNCTION IS A FUNCTION WITHIN A FUNCTION TO FORM A NEW FUNCTION. OR SIMPLY ~~IF YOU TAKE~~ A COMPOSITION FUNCTION HAS A MOTHER FUNCTION AND A ANOTHER FUNCTION WITHING IT. IN THE EXAMPLE OF A CHURCH AND THE PEOPLE IN THE CHURCH. THE CHURCH ALONE IS A FUNCTION ON ITS OWN, SO ARE THE CONGREGATION MEMBERS WHEN NOT IN THE CHURCH. WHEN THE CONGREGATION MEMBERS ENTER THE CHURCH, THEY FORM A NEW FUNCTION OF CONGREGATION MEMBERS IN THE CHURCH.
B206

Figure 5.25: Experimental group's responses on the connection of inverses and compositions of functions to the real world

5.8.3.4. Formulation of inverses and compositions of functions (D4)

The students were also asked to formulate and graphically interpret inverses and compositions of functions and to come up real world examples that resemble inverses and compositions of functions (see section 5.8.2.4.).

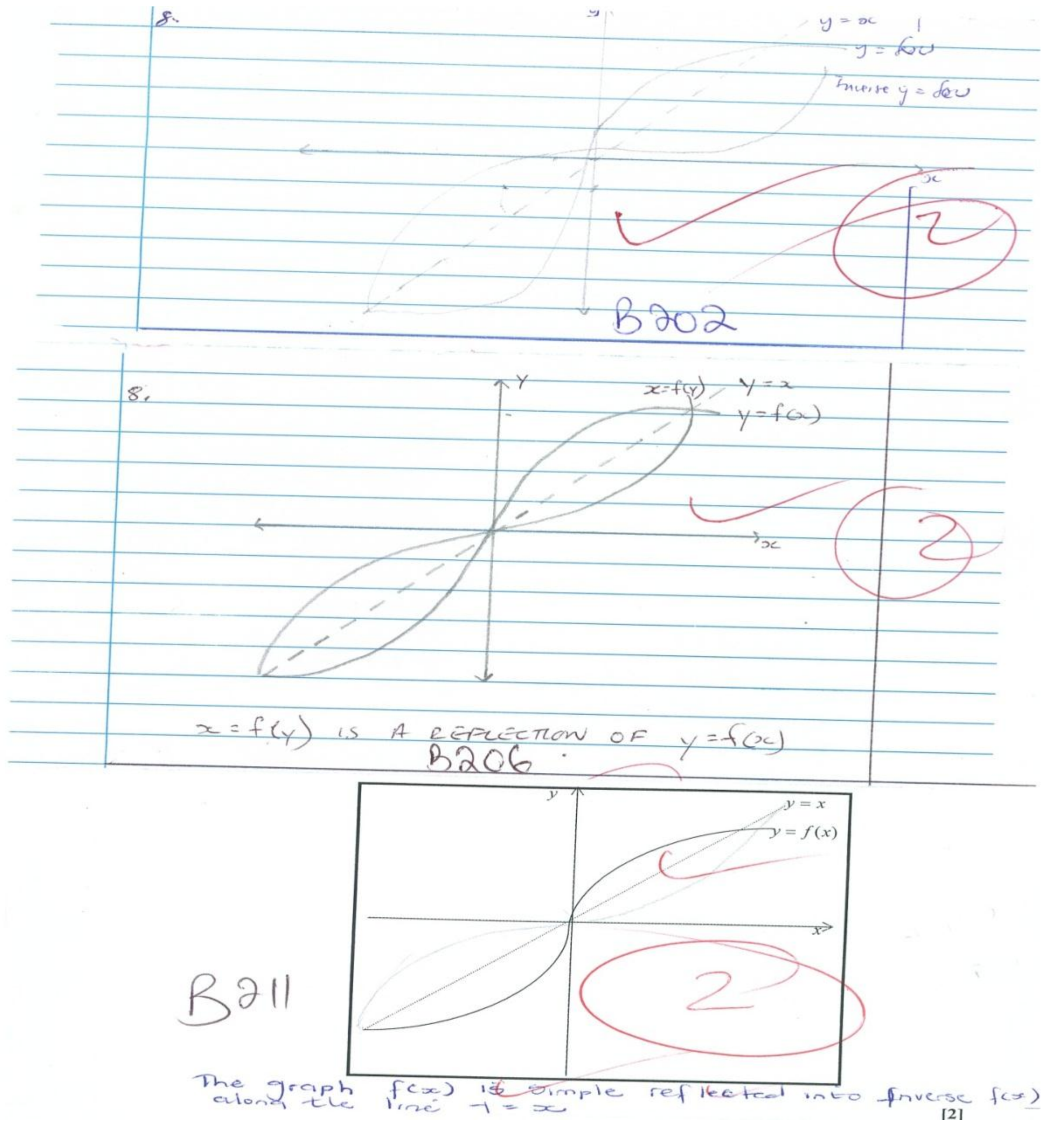


Figure 5.26: Experimental group responses on the formulation of inverses and compositions of functions

-No idea –B238

With regard to composition inverses and compositions of functions, students were given a real life scenario described earlier (see section 5.4.2).The following sample of responses was obtained.

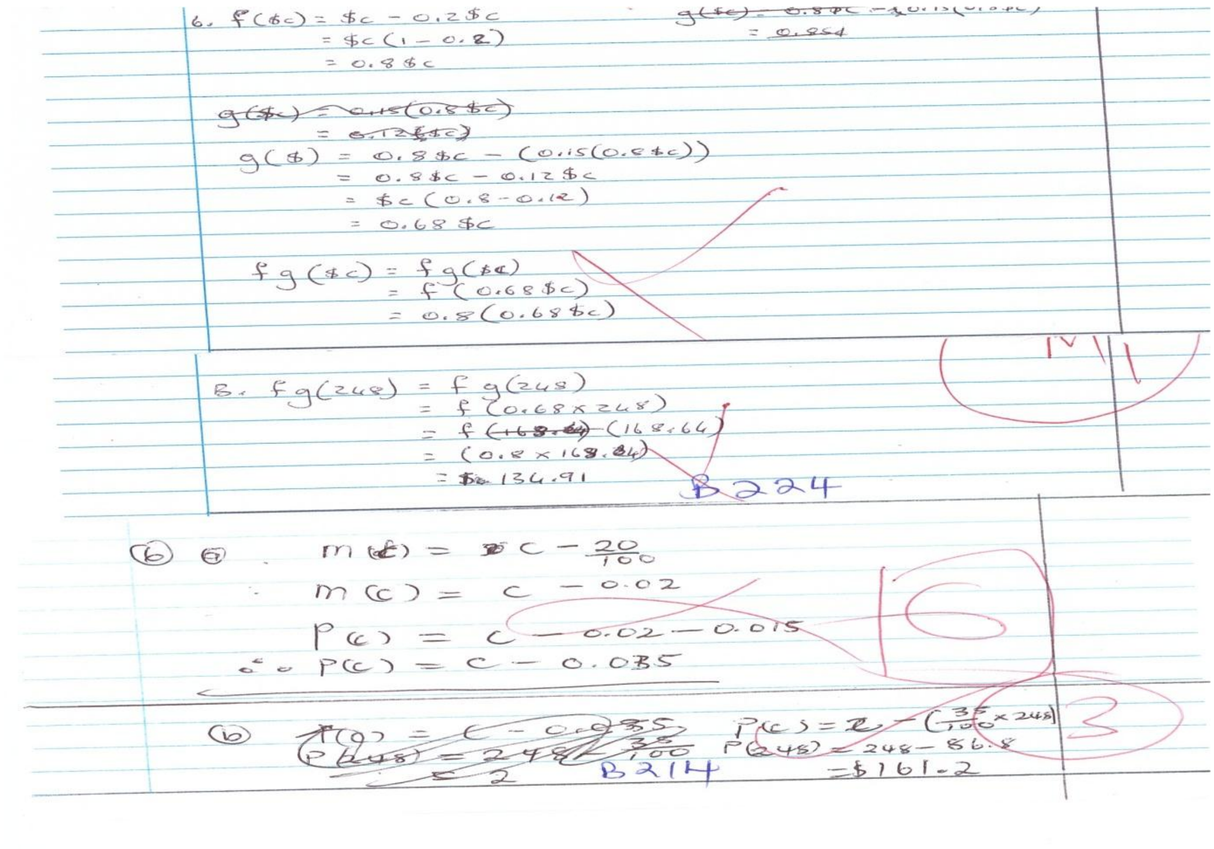


Figure 5.27: Experimental group’s responses on real life scenarios involving inverses and compositions of functions

5.8.3.5. Manipulation of abstract mathematical problems involving inverses and compositions of functions (D5)

The post-test also assessed the students’ ability to find inverses and compositions of functions as part of their experiences with inverses and compositions of functions. The students were given the same two inverses and compositions of functions as before (see section 5.8.2.5) and the following sample responses were obtained:

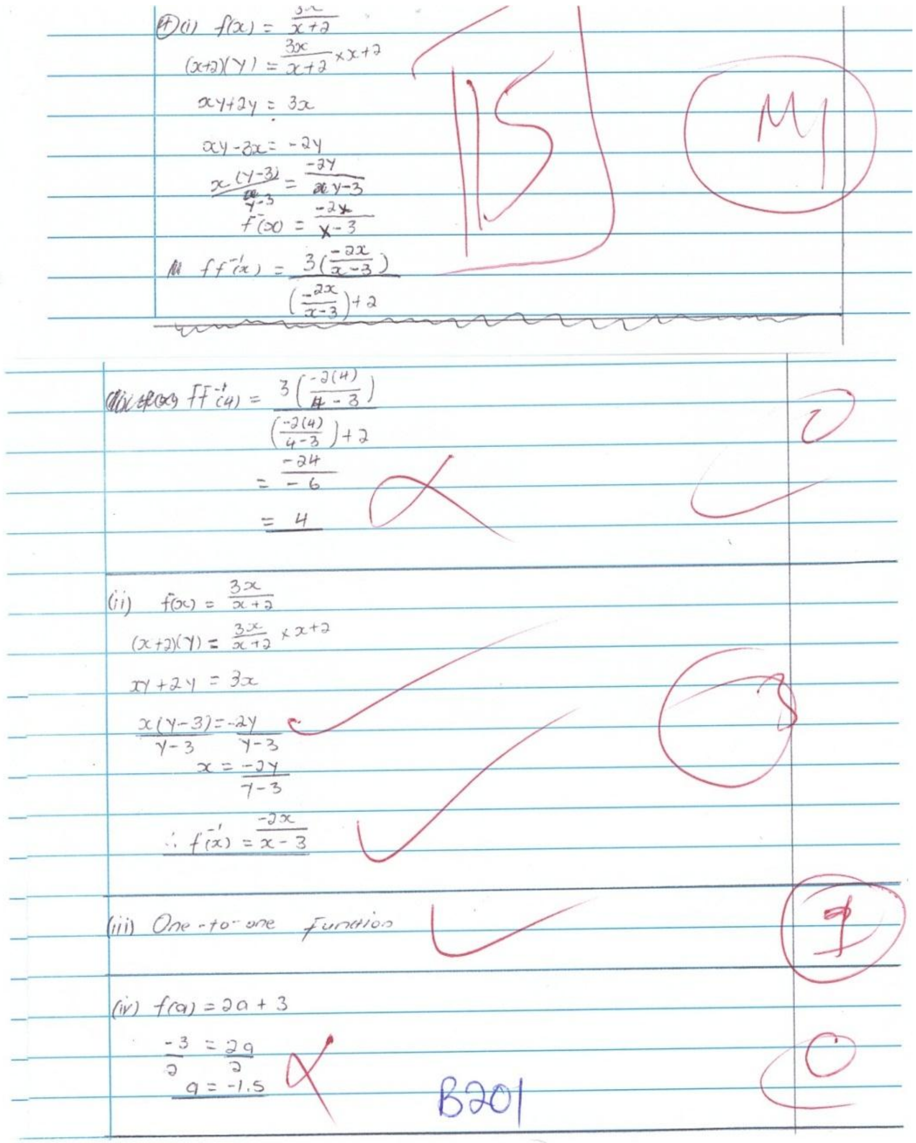


Figure 5.28: Experimental group's responses on the manipulation of abstract mathematical problems involving inverses and compositions of functions.

c) *No idea*

d) *No idea*

-B223

5.9 COMPARATIVE ANALYSIS OF THE POST-TEST RESULTS FOR THE COMPARISON AND THE EXPERIMENTAL GROUPS

After the teaching phase using the two teaching approaches, lecture and PBL, the students in both groups had a better understanding of the definition of a function (D1), from a mathematical perspective depending on their different mathematical backgrounds from high school. Refer to Table 5.9 below for the comparative analysis.

Table 5.9: Comparison of experimental and comparison group's performance by HLT domain in the post-test

Domain	Hypothetical Learning Trajectory Domain	Domain Total Marks	% weight	post-test % pass for X_1group	Variance X1	post-test % pass rate for X_2group	varianceX2	Z test for result for independent groups	Critical value of Z for α=0.05 and	Decision on null hypothesis
D1	Understanding of the definition of a function.	7	10.0	82.9	79.5	78.6	157.1	1.768	1.96	Accept H_0
D2	Representation of a function.	10	14.3	80.3	91.7	60.3	147.8	8.187	1.96	Reject H_0
D3	Ability to connect inverses and compositions of functions to the real world.	28	40.0	82.1	100.0	77.6	89.6	2.067	1.96	Accept H_0
D4	Ability to formulate the inverses and compositions of functions.	14	20.0	71.2	57.9	71.1	106.6	0.024	1.96	Accept H_0
D5	Manipulation of abstract mathematical problems involving inverses and compositions of functions.	11	15.7	80.7	73.0	78.4	144.4	0.9849	1.96	Accept H_0
	Total	**70**	**100**	**79.5**		**72.0**				

Table 5.9 above show that at 95% students' performances for the experimental group and the comparison group for HLT domains D1, D3, D4 and D5 were not significantly

different. However, a significant difference was noted for domain D2 where the null hypothesis was rejected in favour of the alternative. These results suggest that the experimental group participants were not in any way better in connecting functions to real world events than the comparison group as anticipated. Participants from the experimental group outweighed the participants from the comparison group only in the representation of functions (see hypothesis test results in the table above).

After experiencing the two different approaches, students from both groups could define a function, composition function and inverse appropriately. There were not many cases from both groups of students who had no idea of what a function is. However, in one or two cases students would confuse the use of domain and co-domain in their definitions of a function. On the other hand those who defined it from the perspective of an input-output also made the same confusion. Between the two groups the definition of a function as a mapping was common and there was a fair demonstration of understanding of the definition of a function. Though some students tried to define a function from the perspectives of domain and co-domain but there was a mix up of ideas on what domain and co-domain were before instruction, but this problem did not continue after instruction for both groups.

After the post-test the students from both groups were able to represent a function as a set of ordered pairs in line with the definition given by Kleiner (2003) (see z test results for D2). However, for this domain the experimental group performed significantly better than the comparison group (see z test results in the table above). The performance of the experimental group might be attributed by their experiences with the real world scenarios during the teaching where they interacted with reality. Gulseren & Beyhan (2014) state that students need to realize that Cartesian product between two sets (finite and infinite) means all the matching between elements. They further emphasise that students need to be able to state the relationship between a relation and a function such that every relation is a subset of the Cartesian product.

With regard to the connections of inverses and compositions of functions to real world (D3), students from both groups were able to demonstrate their understanding of inverses and compositions of functions on the scenarios which were given. On this domain there was no significant difference in the performance of the experimental group and the comparison group (see the z test decision rule for D3 in the table above). However, there were some cases of students who could not get the sense of the question and could not even make an attempt to apply their previous knowledge of inverses and compositions of functions to solve problems on the scenarios presented. Dubinsky and Wilson (2013) state that the study of inverses and compositions of functions in mathematics is pivotal in the sense that inverses and compositions of functions equip students with proper skills needed to solve problems in calculus and other related concepts. Therefore, students who struggled to extract the mathematics in the given scenarios could not make sense of the questions and would risk performing poorly in pre-calculus in future.

The formulation of inverses and compositions of functions (D4) was fairly done by both groups. However, the aspect of formulating the inverse of a composition function was poorly done by both groups (see decision rule for the z test in for D4). Formulation of the inverse of a one-to-one function requires students' prior knowledge of subject of the formula and solving equations. This poor performance could be a result of the FP students' inability to manipulate algebraic expressions which are the cornerstone of equation solving. Before the intervention students in both groups lacked the flexibility framework for understanding inverses and compositions of functions which Kimani (2008) contends that it allows students to view inverses and compositions of functions from different perspectives, symbolic, verbal or tabular form. Kimani further states that inverses and compositions of functions are central to many branches of mathematics and if a student has a deep understanding of inverses and compositions of functions, he will be able to quickly make mathematical connections that are needed in understanding scientific and other related concepts.

Another important comparison to make is the dimension of lesson focus which was employed when teaching the two groups. The comparison group mostly concentrated on more abstract algebraic function concepts with little focus on how these are related to the real world (D3). However, the few real-world problems which the comparison group worked through had an impact because there was no noticeable difference in performances between the groups on this domain performance. As a result of that students from both groups ended up having the versatility and adaptability of understanding inverses and compositions of functions as referred to by Kimani (2008) (see section 3.3.1). The experimental group had the privilege of applying their understanding of inverses and compositions of functions to real-world examples and it was envisaged that these experiences would enable them to have a deeper understanding of inverses and compositions of functions than their comparison group counterparts. But the z test carried out on this domain showed no significant different in students performances. It was presumed that the PBL group could clearly give relevant examples and explanations of applications of inverses and compositions of functions based on their experiences with the teaching approach. However the employed z test showed no significant difference in students performances on this domain (see z test decision rule above).Participants from both groups fairly developed the flexibility approach (see section 3.3.1.) that gave them a relatively deep understanding of inverses and compositions of functions and were able to solve complex problems related to inverses and compositions of functions and their dynamic forms as stated by Even and Bruckheimer (2008) though their performances were not significantly different.

It can be concluded that the use of Problem-Based Learning in a SFP did not produce significant performance differences though there was a higher level of academic motivation for some domains, which maintained students' attendance at 100%, and students were actively participating in learning and kept on questioning throughout the classes. However, based on these findings it can be concluded that the role that the conventional teaching approaches like the lecture method cannot be undermined in the teaching and learning of compositions and inverse of functions in a SFP

5.10 ANALYSIS OF THE QUANTITATIVE RESULTS FROM THE QUESTIONNAIRE FOR THE COMPARISON GROUP

The students in the two groups were assessed on how they perceived the helpfulness of the employed group activities, the effectiveness of the employed tutorial groups, their level of satisfaction with regard to the employed teaching method, resource adequacy, the effect of the use of real life examples in the teaching, adequacy of the conditions employed to support a particular teaching method and their level of understanding of taught content as a result of a particular teaching method. This section presents the questionnaire results for the comparison group. The full questionnaire is given in Appendix K.

5.10.1. The helpfulness of the employed tutorial sessions.

For the comparison students the main purpose of the questionnaire was to find out how they experienced the lecture method in the teaching and learning of inverses and compositions of functions. As students experienced their learning using the lecture method, this item of the questionnaire intended to find out how helpful the particular teaching method was, Table 5.10 presents the students' responses to the item statement *"Tutorials group were helpful to make me understand inverses and compositions of functions"*.

Table 5.10: Comparison group students' responses to the helpfulness of tutorials in the understanding of inverses and compositions functions

	Frequency	**Percentage**
Strongly Agree	8	20.0
Agree	21	52.5
Disagree	8	20.0
Strongly Disagree	3	7.5
Total	**40**	**100.0**

Comparison group students who were taught using the conventional lecture method also had tutorials with group activities in which they worked through different problems. Based on the table above it can be seen that eight (20.0%) of the students strongly agreed that the tutorial sessions were helpful, whereas 21 (52.5%) agreed, eight (20.0%) disagreed, and finally three (7.5%) strongly disagreed that the tutorials were helpful. Combining those who agreed and strongly agreed to form one category AGREED, and those who disagreed and strongly disagreed to form one category

DISAGREED it can be seen that 29 (72.5%) found tutorial sessions to be helpful in the teaching and learning of inverses and compositions of functions, 11(27.5%) did not find tutorials to be useful. The use of tutorials in this study was a new thing. Under normal circumstances FP students do not use tutorials therefore, the 11 students who did not find tutorials not to be helpful could have said so because they are not used to them, and yet 29 students who felt those tutorial sessions were helpful.

5.10.2. Comparison group students' perception of the effectiveness of their group activities.

To find out how effective the comparison group students perceived the employed group work the questionnaire item asked the extent to which they agreed or disagreed with the statement *"Working in groups meant that you learnt from one another when doing inverses and compositions of functions".* Figure 5.31 below shows the comparison group students' results on this statement.

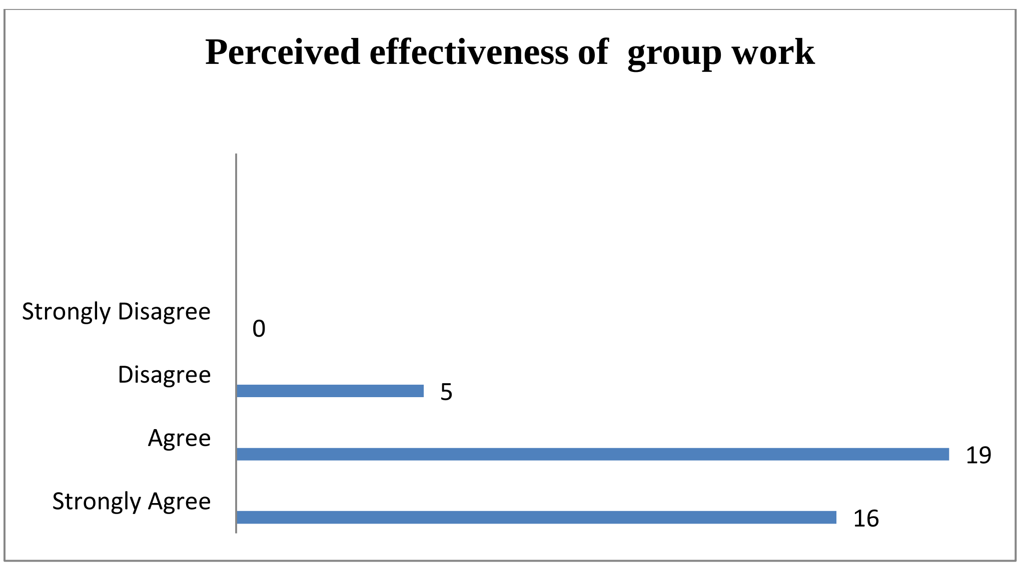


Figure 5.29: Comparison group students' perceived effectiveness of group work

On the effectiveness and the consequent meaning of group work among the comparison group members, Figure 5.31 shows that 16(60.0%) strongly agreed, 19(47.5%) agreed and five (12.5%) disagreed and none strongly disagreed with the statement that group work meant learning from one another during the teaching and learning of inverses and compositions functions. The researcher then combined those who agreed and those who disagreed to form one category AGREED and those who strongly disagreed and disagreed to form one category DISAGREED. It can be seen that 35(87.5%) agreed that group work meant learning from one another among the comparison group members and five (12.5%) did not perceive group work as meaning to learn from one another.

5.10.3. Comparison group students' perceived employment of a student-centred approach during the teaching of inverses and compositions of functions.

This questionnaire item tried to measure the students' assessment of whether a student-centred approach was employed during the teaching and learning of inverses and compositions of functions. Figure 5.32 below shows the comparison group results on this aspect. The students were asked to show the level of agreement/disagreement with the statement: "*A student-centred approach was used during the teaching of inverses and compositions of functions*"**.** Figure 5.32 shows the students' responses on this assertion.

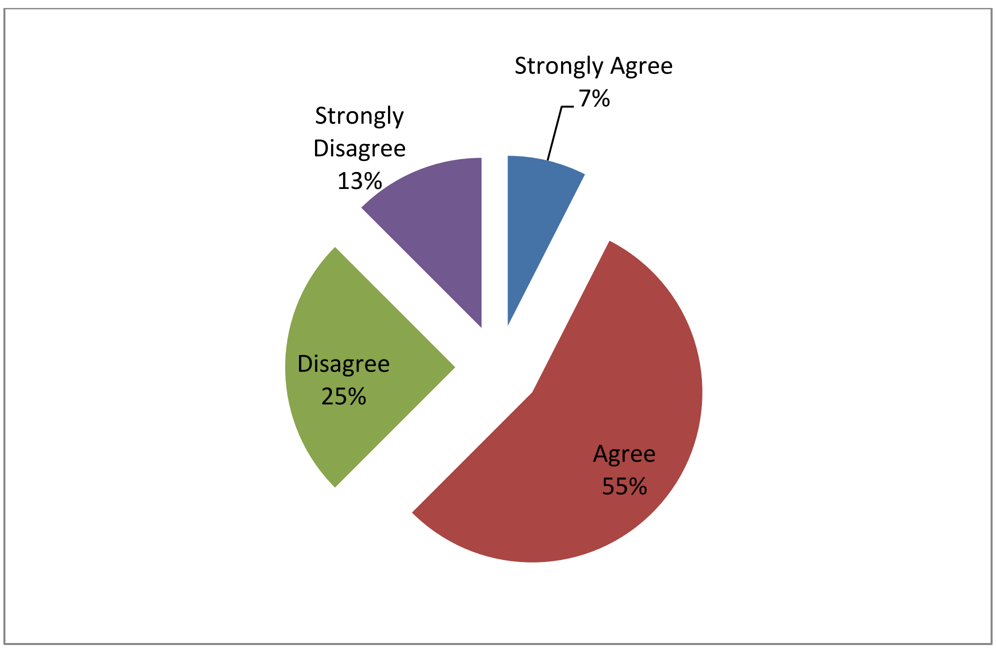


Figure 5. 30: Level of comparison group agreement with the employment of a student-centred approach

Figure 5.32 above shows that three (7.0%) of the students strongly agreed, 22(55.0%) agreed, 10(25.0%) strongly disagreed and five (13.0%) strongly disagreed with the employment of a student-centred approach. The researcher then combined those who agreed and strongly disagreed to form one category AGREE, and those who disagreed and strongly disagreed to form one category DISAGREE. It can be seen that 25 (62.0%) agreed and 15 (38.0%) disagreed with the statement that a student-centred approach was employed in the teaching of inverses and compositions of functions.

5.10.4. Comparison group students' satisfaction with problem solving during the teaching and learning of inverses and compositions of functions using the traditional lecture.

As comparison group students engaged in problem solving in their different groups, the study tried to measure the level of satisfaction in the problem solving which was employed during the teaching and learning of inverses and compositions of functions. Figure 5.33 below shows the perceptions of the comparison students on this attribute. The students were asked to show their level of agreement with the statement; "*I learnt problem solving as much as I would have done in a normal conventional lecture when I was doing inverses and compositions of functions*."

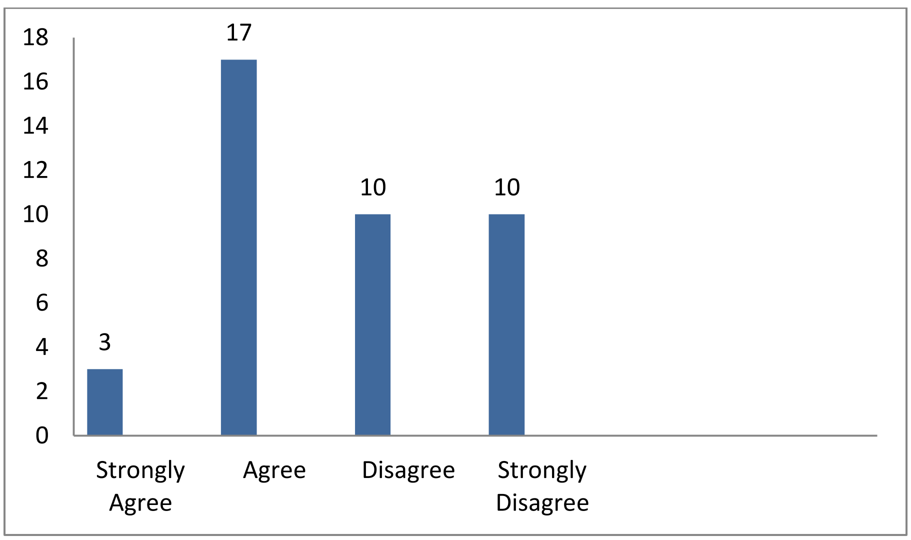


Figure 5.31: Comparison group students' satisfaction with problem solving

Figure 5.33 above shows that three (7.5%) of the students strongly agreed, 17(42.5%) agreed, 10 (25.0%) disagreed and 10 (25.0%) strongly disagreed with the statement that they learnt problem solving as much as they would have done in a normal conventional lecture when they were doing inverses and compositions of functions.

Combining agreed and strongly agreed to form a new category AGREED and combining disagree and strongly disagree to form a new category DISAGREED, it can be seen that 20 (50.0%) of the students agreed and 20(50.0%) disagreed with the statement that they wouldn't have learnt problem solving in a conventional lecture as much as they did.

5.10.5. Comparison group students' evaluation of the sufficiency of the resources provided during the teaching and learning of inverses and compositions of functions.

This item of the questionnaire tried to find out how the comparison students experienced the sufficiency of resources during the teaching and learning of inverses and compositions of inverses and compositions of functions. To find out if the resources were sufficient or inadequate the comparison group students were asked to show their level of agreement with the statement: *"Considering the material I had, when I was doing inverses and compositions of inverses and compositions of functions I few this material was sufficient to make me understand inverse and compositions of functions."* Table 5.11 below shows the results of the students' responses.

Table 5.11: Comparison group students' perceptions of the adequacy of the materials and resources provided during lessons and tutorial sessions

	Frequency	Percentage
Strongly Agree	4	10.0
Agree	18	45.0
Disagree	14	35.0
Strongly Disagree	4	10.0
Total	40	100.0

Table 5.11 shows that four comparison group students (10.0%) strongly agreed, 18 (45.0%) agreed, 14 (35.0%) disagreed and four (10.0%) strongly agreed with the statement that they would have had as much of the provided material in a conventional

lecture. Combining those who agreed with those who strongly agreed to form one category AGREED and those who disagreed or strongly disagreed to form one category DISAGREED, it can be noted that 22(55.0%) agree with the statement whereas 18 (45.0%) disagree with the statement.

5.10.6. The effect of using real life examples in the teaching and learning of inverses and compositions of functions

The main focus of this questionnaire item was to find out how comparison students experienced use of real world examples in problem solving involving inverses and compositions of functions. Figure 5.34 below presents the students' responses on their level of agreement/disagreement with the statement *"Focusing the teaching of inverses and compositions of functions on real life made the topic more interesting and relevant*"**.**

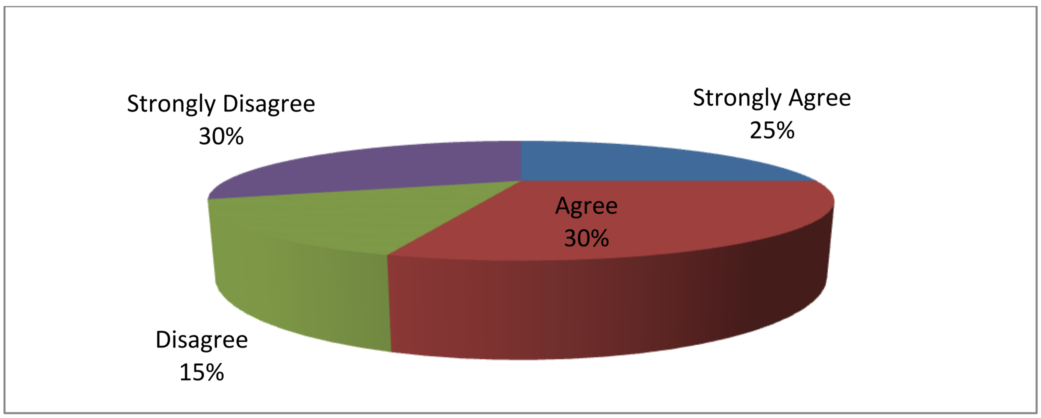


Figure 5.32: The meaning of the use of real life problems

Figure 5.34 shows that 10 students (25.0%) strongly agreed, 12 (20.0%) agreed, six (15.0%) disagreed and 12 (30.0%) strongly disagreed with the statement that focusing the teaching of on real world problems made the topic more interesting and relevant. The researcher then combined the numbers of students who agreed and strongly agreed to form one category AGREED and those who disagreed and strongly disagreed to form one category DISAGREED. It can be noted that 22 (55.0%) agreed and 18 (45.0%) disagreed with the asserted meaning of the use of real life problems.

5.10.7. Adequacy of the created learning conditions to support the lecture method during the teaching and learning of inverses and compositions of functions

To find how the comparison group students felt about the sufficiency of the conditions created for the lecture method in the teaching and learning of compositions and inverses of functions. The students were asked the extent to which they agreed or disagreed with the statement *"Sufficient conditions were created to support lecture methods during lessons and tutorials for inverses and compositions of functions"*. Figure 5.35 below shows the students' results on this statement.

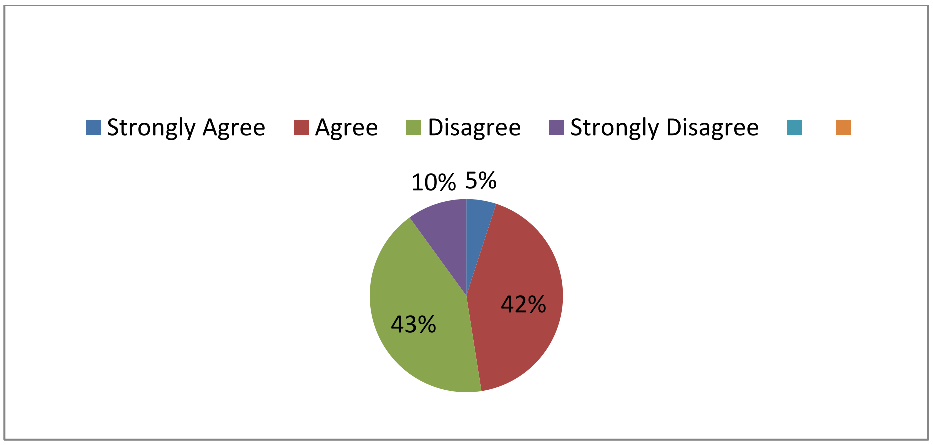


Figure 5.33: Comparison group students' responses on the adequacy of the created conditions to support the lecture method

The comparison group students were asked on the conduciveness of the set conditions to support the use of the lecture method in the teaching of inverses and compositions of functions. This item on conditions was included in order to access whether the students felt that they were well supported in terms having all that they felt they needed to make their teaching and learning of inverses and compositions of functions more productive the figure shows the students' responses on this issue based on their experiences. Figure 5.45 shows that two (5.0%) strongly agree, 17(43.0%) agreed, 17(43.0%) disagreed and finally four (10.0%) strongly disagreed with the statement that sufficient conditions to support the lecture method were created. The researcher then combined agreed and strongly disagreed to form one common category AGREED and also combined the category strongly disagree and disagreed to form a new category

DISAGREED. It can be noted that 19(48.0%) agreed whereas 21(52.5%) disagreed with the statement.

5.10.8 Assessment of the comparison students' understanding after experiencing the lecture method in the teaching and learning of inverses and compositions of functions

After the comparison group students' experienced their teaching using the lecture method, this study tried to assess whether they understood inverses and compositions of functions. Students were asked their level of agreement/disagreement with the statement" *I now understand inverses and compositions of functions*". Table 5.12 below show the results of the students' results to this statement.

Table 5. 12: Assessment of comparison group students' understanding of inverses and compositions of functions

	Frequency	Percent
Strongly Agree	6	15.0
Agree	11	27.5
Disagree	11	27.5
Strongly Disagree	12	30.0
Total	**40**	**100.0**

Table 5.12 shows that six comparison group students (15.0%) strongly agreed, 11 (27.5%) agreed, 11 (27.5%) disagreed and finally 12 (30.0%) strongly disagreed with the statement that they now understand inverses and compositions of functions after their lecture approach experiences. The researcher then combined the students' who agreed and strongly agreed to form another category AGREED and those who disagreed and strongly disagreed to form another category DISAGREED. It can be seen that 17 (42.5%) agreed and 23 (57.5%) disagreed with the statement.

5.11 ANALYSIS OF THE QUANTITATIVE RESULTS FROM THE QUESTIONNAIRE FOR THE EXPERIMENTAL GROUP

5.11.1 The helpfulness of the tutorial sessions.

This item of the questionnaire focused on the experimental students' experiences of PBL in the teaching and learning of inverses and compositions of functions. Specifically, it tried to find out how helpful and effective the PBL approach was in the teaching and learning of compositions and inverses of functions. Figure 5.36 presents the students' responses on the statement *"Tutorials group were helpful to make me understand inverses and compositions of functions"*.

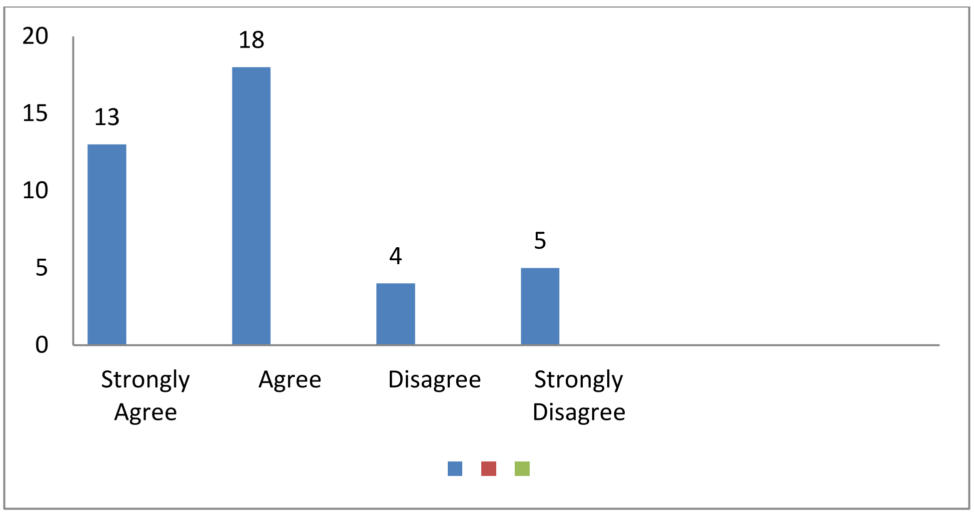


Figure 5. 34: Experimental group students' responses to the helpfulness of PBL tutorials in the understanding of inverses and compositions of functions

Figure 5.36 shows that 13 (32.5%) strongly agreed, 18 (45.0%) agreed, four (10.0%) disagreed and five (12.5%) strongly disagreed with the statement that tutorial sessions were helpful. The researcher the combined the category agreed and strongly agreed to form one category AGREED and also combined disagreed and strongly disagreed to form one category DISAGREED. It can be noted that 31 (77.5%) of the students agree and nine (22.5%) disagree with the statement that tutorial sessions were helpful.

5.11.2. Experimental group students' perceptions of the effectiveness of the PBL group work activities

To find out how effective the experimental group students perceived the group work in the teaching and learning of compositions and inverses of functions, the questionnaire item asked the extent to which the students agreed or disagreed with the statement *"Working in groups meant that I learnt from other students in doing inverse and composition of function"***.** Figure 5.37 below shows the students' results on this statement.

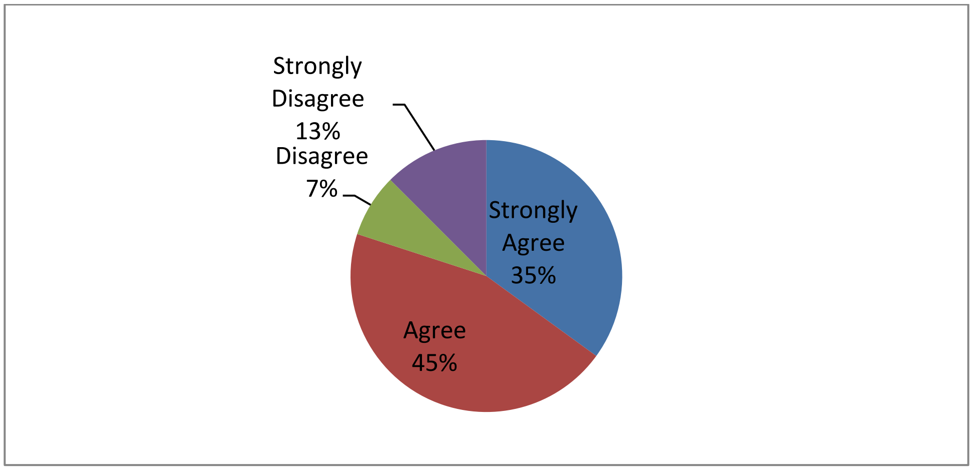


Figure 5. 35: Effectiveness of the employed group work

Figure 5.37 shows that 14 (35.0%) strongly agreed, 18 (45.0%) agreed, three (7.0%) disagreed and five (13.0%) strongly disagreed with the assertion that the group work meant that students would learn from one another in doing inverses and compositions of functions. The researcher then combined those who agreed and strongly agreed to form one category AGREED and those who disagreed and strongly disagreed to form one category DISAGREED. It can be noted that 32 (80.0%) agreed and eight (20.0%) disagreed on the effectiveness of the employed group work.

5.11.2 Experimental group's perceived sufficient guidance during the teaching of inverses and compositions of functions.

Another experience which the study tried to explore was the students' assessment of whether a student-centred approach was employed during the teaching and learning of inverses and compositions of functions. Figure 5.38 below shows the experimental group's responses. As with the comparison group, the students were asked to show the level of agreement/disagreement with the statement: *"Sufficient guidance was given by the lecturer during the teaching of compositions and inverses of functions"*.

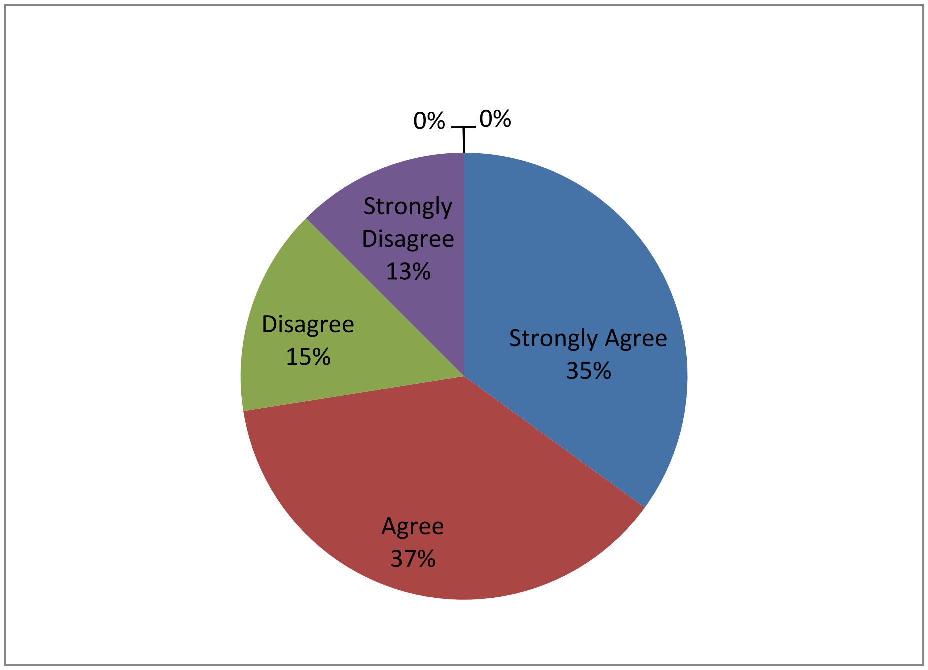


Figure 5.36: Level of agreement with the sufficiency of the lectures' guidance during the PBL sessions

Figure 5.38 shows that 14 (35.0%) strongly agreed, 15 (37.0%) agree, six (15.0%) disagreed and five (13.0%) strongly disagreed with the assertion that a student centred approach was employed during the teaching of inverses and compositions of functions. The researcher then combined those who agreed and strongly agreed to form one category AGREED and those who disagreed and strongly disagreed to form one category DISAGREED. It can be seen that 29 (72.5%) agreed whereas 11 (27.5%) disagreed with the statement.

5.11.3 Experimental group students' satisfaction with problem solving during the PBL teaching and learning of inverses and compositions of functions.

As experimental group students engaged in problem solving using the PBL approach, the study also tried to measure the level of satisfaction with the approach employed during the teaching and learning of the said inverses and compositions of functions.

Figure 5.39 below shows the perceptions of the students on this attribute where they were asked to show their level of agreement/disagreement with the statement. Also, indicate percentages of the collapse categories for this item: *"I learnt problem solving as much as I would have done in a normal conventional lecture when I was doing inverses and compositions of functions."*

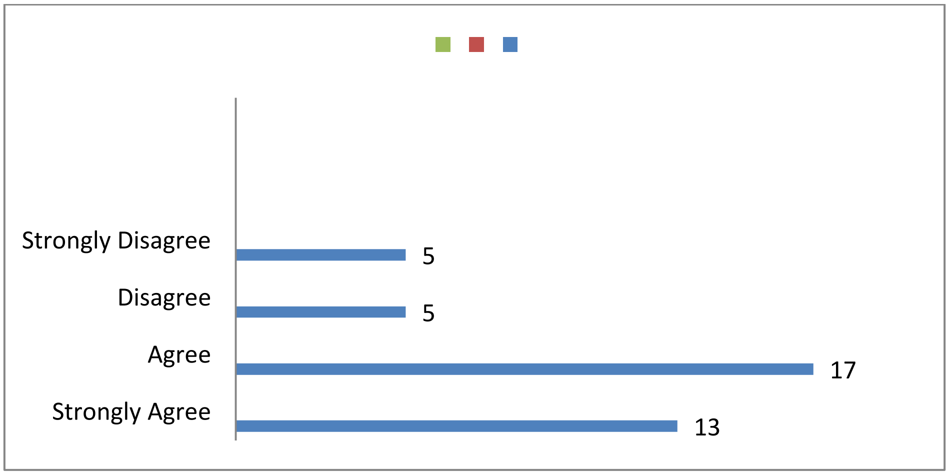


Figure 5.37: Experimental students' satisfaction with PBL problem solving

It can be noted from Figure 5.39 that 30 (75.0%) agreed whereas 10 (25.0%) disagreed with the assertion, 13 (32.5%) strongly agreed, 17 (42.5%) agreed, five (12.5%) disagreed and finally five (12.5%) strongly disagreed that they learnt problem as much as they would have done in a conventional lecture when they were doing inverses and compositions of functions The researcher then combined the category agreed and strongly agreed to form one category AGREED and the category disagreed was also combined with strongly agreed to form one category DISAGREED.

5.11.4 Experimental group students' evaluation of the sufficiency of the resources provided during the teaching and learning of inverses and compositions of functions.

This study also tried to find out how students experienced the change in resources provided during the teaching and learning of the inverses and compositions of functions. To find out if the resources were adequate. Students were asked to show their level of agreement with the following statement: *"Considering the material I had, when I was* *doing of inverses and compositions of functions I think I would have had as much in a conventional lecture."*

Table 5.13: Experimental group students' perceptions of the adequacy of the materials and resources provided during lessons and tutorial sessions inverses and compositions of functions

	Frequency	Percentage
Strongly Agree	9	22.5
Agree	22	55.0
Disagree	6	32.5
Strongly Disagree	3	7.5
Total	**40**	**100.0**

Table 5.13 shows the results of the students' responses and shows that nine (22.5%) students strongly agreed, 22 (55.0%) agreed, six (32.5%) disagreed and finally three (7.5%) strongly disagreed with the assertion that sufficient conditions were provided materials that supported the implementation of the PBL. The researcher then combined the categories agreed and strongly agreed to form one category AGREED and the category disagreed and strongly disagreed to form one category DISAGREED. It can be noted that 31 (77.5%) agreed whereas 9(22.5%) disagreed with the statement that the conditions under which PBL were implemented were sufficient.

5.11.5 The effect of using real life examples in the teaching and learning of inverse and compositions of functions

As with the case of comparison group students' inverses and compositions of functions the main focus of this questionnaire item was to find out how comparison students experienced use of real world examples in problem solving involving inverses and compositions of functions. Table 5.14 below presents the students' responses on their level of agreement/disagreement with the statement *"Focusing the teaching of inverses and compositions of functions to real life made the topic more interesting and relevant".*

Table 5. 14: The meaning of the use of real life problems

	Frequency	Percent
Strongly Agree	8	20.0
Agree	19	47.5
Disagree	9	22.5
Strongly Disagree	4	10.0
Total	**40**	**100.0**

Table 5.14 shows that eight (20.0%) of the students strongly agreed, 19 (47.5%) agreed, nine (22.5%) disagreed and four (10.0%) strongly disagreed with the statement that focusing the teaching of inverses and compositions of functions to real life problems made the topic more interesting, relevant and meaningful. The researcher then grouped agree and strongly agree to form one common category AGREED, disagreed and strongly disagreed also formed one category DISAGREED. It can be noted that 27 (67.5%) of the students agreed whereas (13) 32.5% disagreed.

5.11.6. Adequacy of the created learning conditions to support the PBL approach during the teaching and learning of inverses and compositions of functions

To find out how experimental group students felt about the sufficiency of the conditions created for the PBL approach in the teaching and learning of inverses and compositions of inverses of functions, the experimental students were asked to choose the extent to which they agree or disagree with the statement *"Sufficient conditions were created to support PBL during lessons and tutorials of inverses and compositions of functions".* Figure 5.40 shows the experimental groups' responses.

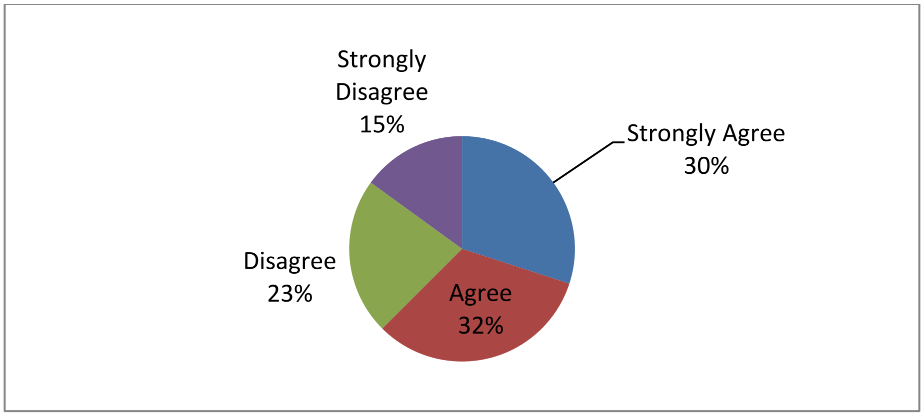


Figure 5.38: Experimental students' responses on the adequacy of the created conditions to support the PBL approach

Figure 5.40 shows that 12 (30.0%) of the students strongly agreed, 13 (32.0%) agreed, nine (23.0%) disagreed and finally five (15.0%) strongly disagreed with the statement. The researcher then combined strongly agreed and agreed to form one category AGREED, strongly disagree and disagreed to form one category DISAGREED. It can be noted that 25 (62.5%) agreed whereas 15(37.5) of the students disagreed with the statement.

5.11.7 Self-evaluation of the experimental students' understanding after experiencing the PBL approach in the teaching and learning of inverses and compositions of functions

After the experimental group students experienced their teaching using the PBL approach, this questionnaire item, as with the comparison group, tried to establish whether they understood inverses and compositions of functions. Students were asked their level of agreement/disagreement with the statement: ***"****I now understand inverses and compositions of functions inverses and compositions of function"*. Figure 5.40 shows the results.

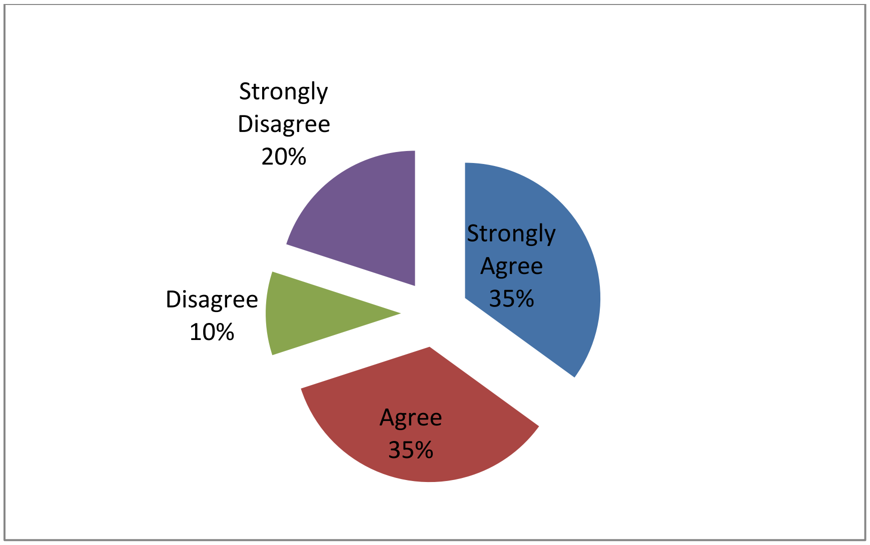


Figure 5. 39: Experimental students' understanding of inverses and compositions of functions

Figure 5.41 above shows that 14 (35.0%) of the students strongly agreed and agreed, four (10.0%) disagreed whereas eight (20.0%) strongly disagreed. The researcher then combine agreed and strongly agreed to form one common category AGREED and also combined strongly disagree and disagreed to form one category DISAGREED, it can be noted that 20 (70.0%) AGREED whereas 12 (30.0%) DISAGREED with the statement that they now understood inverses and compositions of functions as a result of their experiences with the PBL approach.

5.11.8 Comparative analysis of the questionnaire results for quantitative data for the PBL and comparison group

The study intended to compare the FP students' experiences with the conventional lecture approach and the problem based learning with regard to: (1) helpfulness of the tutorial sessions which were rendered –B3, (2) what was meant by group work from the perspectives of the students in the two groups-B4, (3) sufficiency of the conditions to support the PBL/Lecture method –B5, (4) extent of implementation of a student-centred approach during the entire teaching process-B6, (5) students' evaluation of the sufficiency of problem solving -B7, (6) sufficiency of resources to support PBL or the lecture method - B8, (7) students' views on the effect of using real life problems - B9, (8)evaluation of whether they understood compositions and inverse of function as a result of their experiences with a particular teaching method-B10*.* Each student's experience will be represented by a particular code B3-B10*,* and these are the codes which will be used in this comparative analysis.

With regard to students' experiences with item B3, the experimental group members found the tutorial sessions to be more helpful than the comparison group members. Barrows (2002) states that in a PBL environment, problems form the organizing stimuli for learning and they create a desire to find out more on a particular topic which will lead to more concentration, endurance, focused attention and more willingness to learn. The comparison group might not have found this more beneficial to them because tutorials were new and they had to acquaint themselves with them. The qualitative findings for the study also revealed that some students from the comparison did not find the tutorials to be effective in their teaching and learning of compositions and inverses of functions. With regard to item B4-the effectiveness of working in groups both groups found this attribute to be helpful. They held the same view that working in groups provided opportunities for them to learn from each other. Kolmos (2006) states that group work in a PBL environment provides an additional opportunity for learners in the learning competencies. Process competencies are those competencies and skills which individuals require over and above those "hard skills" that are related to the minimum requirements that enable an individual to practice in mathematics. During the employment of the group work and tutorials the comparison group members learnt to work together, sharing ideas and collaborating in academic information sharing. Apart from that the group tutorials also helped the students to improve on their communication skills since they had the opportunity to discuss their group findings.

With regard to item B5 - availability of the necessary conditions to support the PBL or Lecture method the comparison group students could not notice any differences in the conditions of their learning. As such those who found the necessary conditions to be available were the same number as those who said the necessary conditions were not available to support the lecture method. On the other hand, more than 90% of the experimental group members were content with the PBL conditions which were set for them. In the lecture environment the teacher is regarded as the "fountain of knowledge" while the students assume the role of inactive recipients of knowledge. Therefore, during the tutorial sessions the comparison group members had to be reminded by the facilitator of doing their group activities from time to time since they expected the lecturer to retain the role that he had always assumed which was not the case with tutorials. However, Schmidt et al. (2011) state that in a PBL classroom tutors are expected to facilitate or activate student learning and to promote effective group functioning by continuously encouraging the group members' active participation, monitoring the quality of the on-going learning and making interventions only when necessary. This characteristic of PBL group work is what might have made the conditions more conducive for learning among the PBL group members as opposed to the comparison group members.

In item B6 about the implementation of a student-centred approach; both groups confirmed that the entire learning was student-centred. Barrows (1996) states that PBL environments must be student centred and problems are used as the vehicle for development of academic and professional problem solving and new information is acquired through self-directed learning.

In item B7 about the adequacy of exposure to problem solving students in the comparison group saw no significant difference in the problems they were solving as they compared them to their conventional lecture. Twenty did not see a difference and 20 saw a difference. This half-half distribution of agreeing and not agreeing among the participants suggests that there was not much change to the students' problem solving as compared to what they usually experience in their normal classroom settings. However, the PBL group had 75% of the students confirming that the problem environment was rich and could not be compared to their usual conventional lecture environment. Belland et al.(2009) argue that in PBL students are asked to put their knowledge to use and to be reflective and self-directed learners. This was probably the experience which the experimental group had during the course of their PBL sessions.

Item B8 of the students' experiences was to explore the sufficiency of the study materials they were using. Half of the comparison group students confirmed that the materials were adequate and the other half did not find the materials to be adequate, whereas for the experimental group found the material to be sufficient enough to support the PBL approach. Downing (2001) states that the role of PBL is to achieve instructional goals, such as promoting students' knowledge transfer making use of knowledge in context, helping students develop cognitive reasoning skills, and self-directed learning skills, and increase the motivation for learning among the students if sufficient resources are provided. Lohman (2004) states that PBL problems should be messy, complex in nature, and should be ill-structured inorder to stimulate real-life experiences. This will help the students to interact with reality rather than battling with abstract mathematical concepts. By providing students with sufficient study materials to necessitate PBL, students are taught to be able to make an intergration of this process and experiances to enhance their problem solving skills and are able to solve real-life problems. It was these real-life experiences which made the students' learning more interesting and realistic in the case of the experimenntal group.

The uniqueness of PBL is that it uses real world problems and sometimes ill-structured problems. In this study the students experienced their PBL and lecture using real world problems to find out views on the true meaning of working with real world problems. The comparison group students did not find the use of real-world examples to be important since only 55% agreed with the statement that real life problems helped them to understand mathematics better and the experimental group had 62.5% who found this experience of working with real world problems to be fascinating. Lohman (2004) emphasises on the need for students to learn from the world's knowledge and accumulated expertise by virtue of their own study and research, just as real practitioners do. PBL is a tool that can help learners to be autonomous thinkers and independent researchers in the real world.

As students experience their particular teaching method, the main purpose was to find out if performance improvement in the teaching of inverses and compositions of functions. Item B10 evaluated if the students' experience with a particular teaching method resulted in performance improvement in the teaching of inverses and compositions of functions. Less than 50% of the comparison group students confirmed a positive change in performance as a result of their experiences with the conventional teaching approach whereas the experimental group reported more than 80% performance improvement resulting from their experiences with the PBL approach. Comparing these findings with the findings at post-test, it can be concluded that the use of the PBL approach improved students understanding of functions.

5.12. ANALYSIS OF THE FOCUS GROUP INTERVIEW RESULTS FOR THE COMPARISON GROUP

The purpose of the focus group interviews was to find out the students' perceptions with regard to the relevance of compositions and inverses of functions as a concept in a topic that determines their academic destinations and also to find out how the PBL approach be best implemented in order to improve FP students' understanding of composition and inverses of functions. The comparison group was meant to be the standard for comparison with the experimental group findings. The focus group interview results will be presented and analysed in terms of the following domains as contained in the interview schedule in Appendix C (1) general teaching and learning of inverses and compositions of functions at the FP, (2) students' perceptions with regard to the teaching of inverses and compositions of functions taught using the conventional approach (3) students personal experiences with inverses and compositions of taught using the conventional approach and (4) recommendations by students based on their personal experiences with the conventional approach.

5.12.1 General teaching and learning of inverses and compositions and inverses of functions

Three students felt that inverses and compositions of functions were a bit challenging as compared to any other topic they have had prior to them. With regard to the general teaching and learning of inverses and compositions of functions, this is what the comparison group had to say:

-The finding of inverses and domains was quite difficult and confusing- C2

-Finding the inverse of a composition function was difficult because it involved a lot of other mathematical processes- C8

-Formulating a composition function from a word problem was also a very confusing concept - C4.

-I was not comfortable with the use of the horizontal line test for one-to-one inverses and compositions of functions- C4.

-Finding a composition function from a set of given exponential, trigonometric and logarithmic inverses and compositions of functions needed a lot of expertise-C10.

The comparison group students were also asked within their multiple experiences, the aspects they enjoyed most in the teaching and learning of inverses and compositions of functions.

-I enjoyed finding the inverse of a function-C5

-I enjoyed group activities involving real-life problems on inverses and compositions of functions-C8

Apart from that, the students were also asked on the relevance and importance of the rendered tutorials since this was something new to them, based on their experiences.

-Group work was very important because that was the only time we could interact meaningfully in an academic discussion as students. I really benefited a lot from them-C10.

-Tutorials were very much helpful because we had time to research for solutions on our on without the teacher leading our discussion-C2

-I enjoyed tutorials because they changed our usual academic tone where we have to listen to the teacher who is always preaching in front-C9.

In addition to that, the students were also asked on the helpfulness of the employed group activities which they had during the teaching and learning of inverses and compositions of functions. This was done to triangulate the questionnaire results.

-Some group members were just waiting to argue on whatever has been said-C5

-Some group members were just sitting listening to other group members so that they can poach answers-C1.

Researcher: what do you think could be done to make sure that all the students are involved in the group activity especially those who would be busy poaching answers from the next group?

-The group leader must keep reminding all the group members to pay attention and be involved-C5.

5.12.2 Questions related to students' perceptions about the teaching and learning of inverses and compositions of function.

With regard to issues of perceptions and share their views and experiences regarding the teaching and learning of inverses and compositions of functions.

-My perception was just positive I did not notice any change except for the introduced tutorials and a few group works during the teaching of inverses and compositions of functions-C4

-I liked the use of tutorials because they allowed each one of us to put our input and when things are clarified we understood better-C6.

- I don't like it when the teacher has to teach for the whole one hour talking while we are quiet but the involvement of group works and tutorials was ok-C7.

The students were then asked about their perceptions on the relevancy of the questions which the teacher used when he was teaching inverses and compositions of functions

-The questions were real and practical and they resembled the real world-C9.

-But word problems were too difficult for me to attempt and to interpret, I don't think I enjoyed them-C8

5.12.3. Questions related to students' personal experiences with inverses and compositions of functions taught using the lecture approach

With regard to what was counted as being the important aspect in the teaching and learning of inverses and compositions of functions, this is what the comparison group had to say:

-The concept inverses and compositions of inverses and compositions of functions is having too many topics involved in it such that one has to master all the other topics beforehand for him/her to comprehend inverses and compositions of functions well – C1.

Researcher: What topics were involved which needed to be mastered in advance?

-Inverses and compositions of functions require one to be good in solving equations and performing substitution and dealing with subjects of formula, and sometimes one need

to understand the drawing of graphs, and also understanding English for him to apply all these concepts in inverses and compositions of functions- C1.

-One needs to know laws of indices, and simplifying expression for him to handle inverses and compositions of functions-C6.

Regarding their experiences with inverses and compositions of functions, the researcher asked the comparison group members how they got to know that they have mastered a particular concept.

-I have to read and re-read on a concept and then apply my knowledge through practicing more problems and checking how my solutions relate/ differ from the given solutions in the text book, that way I can confirm if I have mastered or not –C8.

-My strategy is that I should know who are those people in my group/class who are better than me, so I always make sure that I cross check whatever I have attempted with them, to see if I am in the right direction-C7.

-I judge my on solutions before I submit for me to see if they are meaningful- C2.

Comparison group students were also asked to state what they thought were the qualities of a good tutorial session from their own perspectives and experiences.

-The lecturer, the students should all work together to come out with solutions to problems, rather than being a teacher dominated tutorial session –C1

-It should start with an individual student alone attempting questions and then share with the group whatever he/she has found, then they come up with group findings which will be shared with the whole class and the discussion of the results go on-C10

-A good tutorial session should make a proper connection of what has been learnt in class with what is being done in the tutorial and the two should be linked.-C4.

-Group sizes should be small, at least four or five students for maximum interaction and easy coordination-C10

-A good tutorial should leave a platform for students to discuss their findings, debate over them and present them to the class and still defend them accordingly-C2.

-Should have a good communication between/ among all the stakeholders involved-C7

On students' experiences the researcher asked them to tell him what they felt were the most challenging aspects when it came to the teaching and learning of inverses and compositions of functions based on their experiences. This is what the students had to say.

-Coming up with a function from long and winding English statement was not easy for me-C5.

- To find the inverse of a composition function was very difficult and confusing-C4.

-Some of the aspects we were doing were different from what is in past exam a paper-C7

-Domains and ranges of inverses and compositions of functions were difficult to formulate- C6

The students were then asked what their roles within the group would be if they were to be put in a tutorial session.

-I have to listen before I present my side of the argument- C8.

-I prefer stating what I think is the correct answer so that when the correct solution is presented I can adjust my way of thinking accordingly –C12.

Students were then asked of the role of the lecture approach in the teaching and learning of inverses and compositions of inverses and compositions of functions.

-To push the syllabus- C3.

-To make the lecturer's work easy and confuse students more – C11.

Researcher: How are the students confused by the lecture method?

-Sometimes the lecturer is just there talking to the chalkboard and the students are struggling to follow what the lecturer is teaching and sometimes he can teach for two hours while students are not following, they get more confused-C9

Then on the issue of how the lecture method can best be implemented in the science bridging course for undergraduate mathematics if it has any role to play in the teaching and learning of inverses and compositions of functions, this is what students had to say.

-Yes it does have a role but we can't have the lecture method every time and again at least mixing it with other methods will make the learning more interesting than lecturer dominated-C6.

-The use of the lecture method with group activities and tutorials like we had can make the learning more interesting and enjoyable –C11.

-I think the teaching method has no problem the problem is what is presented and how its presented, lecturers always speak in x-y language which can end up boring but if some reality and tangible case studies are incorporated then teaching and learning it can be ok - C3.

Students were then asked on how they perceived the grouping which was done to them during the tutorial sessions.

-When you are in group you learn better without getting bored, you feel involved and unisolated-C12.

-Learning in a group is real learning for me unlike learning alone sometimes I am struggling alone to make sense of what is being taught but in a group I grasped a lot-C9.

-Group learning is good if you are not learning with selfish group members who just consider their own benefit -C10.

5.12.4 Recommendations by the comparison students based on their experiences with the lecture method.

Students were then asked to give their recommendations based on their personal experiences with the lecture method. On the improvements students felt need to be made at FP in the teaching and learning of inverses and compositions of functions, this is what the students had to say.

-Lecture method need to be incorporated with productive tutorial sessions-C11

-Students should be involved more in the learning than having the lecturer preaching to quiet students who are not contributing anything to their learning-C5

-Lecturers should try to include real life examples in their teaching to make it more relevant and exciting-C8

Students were then asked to share their best study strategies which improved their understanding of inverses and compositions of functions during their experiences with the lecture method. Here is what some of them had to say.

-I used to interact with people who like school whom I know perform better than me and that is how I improved my performance. –C6.

-I don't believe in group learning I believe in doing things on my own where I don't understand I have to consult the lecturer-C4.

-I had to make connections with previous topics and join them with the new the content for me to understand- C10

-I had to constantly practice the questions which were given to me and I searched for more information in the library for me to learn more-C8.

-I read more around any topic the teacher has taught for me to get a broader understanding of the topic- C6.

The comparison group students were then asked on the advice they would give to anyone aspiring to join FP that day and this is what they had to say:

-I will tell them that mathematics is not as easy as they might think it is-C4.

-Not to miss mathematics lessons ever if they are to pass-C7

-Never to go to sleep with work that has not been understood – C11.

-When the lecturer has taught a concept in mathematics study a bit further around the same concept for you to be safe –C6.

-Mathematics need more practice every day-C10.

-Whenever you have not grasped a concept make an effort to follow-up on it before the next concept-C11.

-Failing while you are in a group is normal at FP so work hard on your own and mix with the right people for you to succeed in mathematics- C12.

5.13. ANALYSIS OF THE FOCUS GROUP INTERVIEW RESULTS FOR THE EXPERIMENTAL GROUP

The purpose of the study was to find out the students' experience with the PBL approach and to triangulate the findings from the questionnaires and observation protocols made.12 students were randomly selected to participate in the focus group discussion for the PBL group. The group interview results will be presented and analysed in terms of the following themes: (1) general teaching and learning of inverses and compositions of functions inverses (2) students' perceptions with regard to the teaching and learning of inverses and compositions of functions using the PBL approach (3) students personal experiences with inverses and compositions of functions taught using the PBL approach (4) recommendations by students based on their personal experiences with the PBL approach.

5.13.1. General teaching and learning of inverses and compositions of functions

Four students felt that the topic inverses and compositions of functions was a bit difficult to comprehend, but the strategy which the teacher employed though it was like one has been thrown in the deep end of a pool, but they managed to understand the topic.

-Determining if a function was one-to-one was a difficult task and needed a lot of hard work-E1

-Finding the inverse of a composition function was a bit tricky because it involved a lot of other mathematical processes- E5

-Forming a composition function which is meaningful was difficult and confusing more especially extracting the mathematics in the English if the problem was a word problem; it was not easy at all- E3.

-I also had misconceptions on what to make the subject of the formula between the given variables when I was extracting the mathematics in a word problem- E12.

-Logarithmic inverses and compositions of functions were difficult to work with especially if one wants to find the inverse of the function-E11

Students were also asked within their multiple experiences with PBL, the aspects they enjoyed most. Here is what they had to say.

-I enjoyed the word problems when it comes to real life applications of inverses and compositions of functions-E6

-I enjoyed finding domains and ranges of inverses and compositions of functions-E7

Students were also asked on the relevance and importance of the tutorials since this was something new to them. Here is what they had to say:

-If you are given a problem to work in a group and you have no idea on where to start from you would learn from the other group members and you get to know things better-E3

-Tutorials were helpful because most of the times when we are given some work to do even in class sometimes we do not do it but now with the introduction of tutorials we had to read because you have to be answerable to the group members and the lecturer. Therefore, tutorials boosted our understanding of inverses and compositions of functions-E2

-Tutorials created a platform for sharing ideas during the tutorial sessions-E8.

The students were also asked on the helpfulness of the group activities which they had during the teaching and learning of inverses and compositions of inverses of functions. Below is what they had to say.

-Some group members were just waiting for answers without thinking-E10.

Researcher: what do you think could be done to make sure that all the students are involved in the group activity?

-I suggest that each student must be given his own problem to work on that's the only way to keep them involved-E10.

Researcher: Is that not going to defeat the purpose of group works which you already highlighted earlier own?

-Yes it might defeat that but at least someone has done something by the end of the tutorial-E10.

-Group discussion on a particular aspect must be done but after that each group member must sit and write his/her own work as he/she understands the problem then the answers can be compared later-E6.

5.13.2. Questions related to students' perceptions about the teaching and learning of inverse and compositions of functions

With regard to perceptions, the students did not understand the meaning of the term inverses and compositions of functions and the researcher had to clarify the definition. This is what the students had to say:

-I prefer you teaching the old way than the new way, I prefer the group and the lecture mixed together-E2

-The employed method was good because some students may feel shy to ask questions from the lecturer who is just preaching but if they are given group activities and tutorials it allows them to interact with the other group members, the only problem we had was that this is our first time to be taught in this way but if this method is to be used every day the students will get used to it and will learn to appreciate it with positive perceptions-E9.

- I prefer that the teacher teaches first and then give work to the students to do; the idea of sharing was good because we learnt more from one another-E12.

-Group work helped me to see that there is more than one way to approach a problem-E8

The students were also asked about their perceptions on the relevance of the questions which the teacher used when he was teaching inverses and compositions of functions. Here is what they had to say:

-The questions which were asked by the teacher were real and practical –E9.

5.13.3. Questions related to students' personal experiences with composition and inverses of functions taught using the PBL approach

With regard to what was counted as being the important aspect in the teaching and learning of compositions and inverses of functions. Below are the students' responses.

-The students need to read and understand the questions well before making an attempt to answer it- E1

-Inverses and compositions of functions tent out to be algebraic and rely heavily on algebraic concepts therefore it is very important for the student to know algebra, subject of formula and substitution. Apart from that, one must be good in extracting the mathematics in a long and winding English statement-E4.

-One needs to know laws of indices for him to handle inverses and compositions of functions-E5.

-One should be a fast thinker when doing PBL because the student does most of the work while the lecturer is just directing-E2

On students' experiences with the PBL approach during the teaching and learning of inverses and compositions of functions, the researcher asked them how they got to know that they have mastered a particular concept. Here is what the students had to say:

-I get to know that I have done it right after I have consulted the teacher to approve what I have written-E7.

-At least I have to try doing difficult questions then I consult my other group members or colleagues to direct me where I have gone wrong-E6.

-I know I am right when I have written answers that make sense- E10.

Students were then asked to state what they thought were the qualities of a good tutorial session from their own perspectives. This is what they had to say:

-It should involve both the lecturer and the students and the lecturer working together, unlike the teacher preaching out answers on the chalkboard while the students are listening, at least students should be given an opportunity to try out something on their own-E9.

-The teacher must also consider the reasoning of the students-E2.

-A good tutorial session should connect what has been taught in class with what has to be leant in the tutorial session-E4.

With regard to students' experiences the researcher asked the students to tell him what they felt were the most challenging aspects when it came to inverses and compositions of functions. This is what the students had to say:

-Coming up with a function that matches what is being asked-E5.

- To formulate and interpret and composition function from a word problem was quite challenging-E4.

-Domains and ranges of inverses and compositions of functions were difficult to formulate especially for inverses and compositions of functions with square roots and logarithmic inverses and compositions of functions- E3

Students were also asked what their roles within the group would be if they were to be put in tutorial session and this is what the students had to say:

-I will be listening to what they say before I say out anything- E6.

-I will explain my perspective of the idea so that if I am wrong I can be corrected by other students E1.

-I will be the writer of what they are saying- E12

-I would beg to differ with whatever they are saying so that as I argue with me they clarify better and I will understand better-E11.

Students were then asked about the role of problem based learning in the teaching and learning of inverses and compositions of functions and this is what they had to say.

-To train you to be a fast thinker- E3.

-Helps you to understand mathematics from a real world perspective- E2.

-It helped me to realize that whatever we are doing in the classroom is practically applicable to the real world-E5.

Then on the issue of how PBL can best be implemented in the SFP if it has any role to play. This is what the experimental group had to say:

-It should be introduced at the beginning of the year, at the onset of Science Foundation-E9

-It is a very important teaching strategy which must also be employed in other subjects because it makes learning more realistic and interesting by its nature-E1.

-Mathematics tends to be boring if it's just numbers every time but with PBL you interact with realities that will help you to be a thinker in the real world- E4.

Students were then asked on how they perceived the grouping which was done to them during the implementation of PBL. This is what they had to say:

-When you are in a group you learn better without boredom-E6.

-If you are learning on your own sometimes the mind can get swayed with useless things but in a group you remain engaged and focused- E9.

-The groups definitely help, not only if you don't know the answer, but hear it from others, you really understand better-E4.

5.13.4 Recommendations by experimental focus group students based on their experiences.

Students were then asked to make recommendations based on their personal experiences with the PBL teaching and learning of inverse and compositions of functions. Here is what they recommended:

-PBL is needed in FP but must be introduced at the beginning of the year-E1.

-Inverses and compositions of functions must be introduced at the beginning of the year- E5

Researcher: Don't you think there was a good purpose to have inverses and compositions of functions in the second semester?

-It was a good purpose because inverses and compositions of functions have many other topics which they rely on-E5.

-It should have been done at the beginning of the year and should be spread throughout the year and should also be implemented in all the other topics in FP-E8.

-PBL helped me to unite concepts- E4.

-Too often when concepts are learnt, two months later they are forgotten, PBL left me with knowledge that I hope will be retained forever-E4.

Students were then asked to share their best study strategies which improved their understanding of inverses and compositions of functions. It was important for the students to share their bests study strategies so that they can inspire their colleagues in improving their own performances too. With regard to study strategies, this is what the experimental group had to say:

-I normally study on my own, but with inverses and compositions of functions we formed a group that I was working with and this greatly improved my understanding of inverses and compositions of functions-E3.

-This group thing was not helping me much so I had to consult the library and did my own self-study and I understood inverses and compositions of functions better-E4.

-I had to study the basic concept underlying inverses and compositions of functions and solved many problems and when I got stuck I asked my friends- E5

-I constantly practiced on the questions which were given to me and searched for more information from the library and internet on questions related to inverses and compositions of functions-E3.

-I had to manipulate the different questions which we were given in class- E6.

-I had to look for more real world problems that resemble function-E1

-I compared inverses and compositions of functions with sets and I would work first what is in the bracket- E10.

-I studied the whole chapter on my own and had to relate what was taught to me last year to the new stuff-E5.

-I have no strategy at all because up to now I still don't understand inverses and compositions of functions-E2

On what advice to give to anyone aspiring to join FP that day, this is what the experimental group students had to say:

-I will tell them that the only way to pass mathematics is to practice more and more-Student A.

-Never miss mathematics lessons-E7

-Make an effort to understand whatever has been taught before the next lesson otherwise you pile work back log on you which you will never manage to cope with- E8.

-They have to read ahead of the lecturer always so that nothing new is introduced before you have not read through it-E6.

-Practice more and more, and must never be too relaxed-E7.

-Grasp the basic concept around a particular idea, and tell them to explore more on the taught idea –E8.

-Never take anything for granted during a lesson, take every lesson seriously- E9.

5.13.5. Comparative analysis of the experimental and comparison groups on the focus group findings.

There are similarities and differences in the way students in the two groups experienced and perceived the teaching and learning of inverse and compositions of functions based on the focus group discussion results. With regard to teaching and learning of inverses and compositions of functions both groups found the formulation of the composition of a function to be a challenging aspect (see C8, C4 and E5). Both groups also found the aspect of determining the inverse of a function to difficult (see C2 and E5). The comparison group further stated that dealing with exponential, logarithmic and trigonometric inverse and compositions of functions was very challenging and confusing (see E11). Both groups found logarithmic inverses and compositions of functions to be difficult especially when it came to finding the domain and range (see C10 and E11). On this note Even & Bruckheimer (2008) state that the interconnection of algebraic concepts make the understanding of functions more difficult since they relay heavily on other topics which should be taught pior to them, inorder to lay the proper base upon which functions will rest.

With regard to what the students enjoyed most during their experiences with their particular teaching method, both groups stated that they enjoyed the involvement of group work where they had to learn from each other (see E3 and C8). Both groups also found the tutorial sessions to be beneficial in making their learning more meaningful (see E6 and C2). Well-coordinated tutorial sessions are able to boost students understanding of any concept being taught in the classroom (Dejan, Kirebi, & Molatu, 2013).The experimental groups further stated that they enjoyed working with real world problems and attested that it really enhanced their understanding of inverses and compositions of functions. The comparison group stated that there were some members of their group who were waiting to be given answers without their contribution. On this aspect, the experimental group members felt that each person must be given his own problem to work on so that they can be accountable to the leader and the facilitator, and the comparison group members felt that the group leader must keep reminding all the group members to remain focused on the group discussion. The experimental group further felt that group discussions on inverses and compositions of functions was good but they suggested that at the end of the discussion each student must make his own write up of the group activity (see E6). On the students' personal experiences with inverses and compositions of functions, both groups felt that inverses and compositions of functions as a topic rely heavily on other basics like change of subject of formula; algebraic manipulations, indices and logarithms and trigonometry etc. (see C1 and E4, E5). These findings concur with Bogdan (2000) cognitive tasks in PBL where he stated that PBL tasks must be set in such a sequential order (remembering that things learned earlier might help with the current task or a problem solving) and metacognitive tasks (monitoring and directing the entire process of problem solving), it should stress more on how to learn about thinking. The students from the two groups appreciated the order and sequencing of topics taught before inverses and compositions of functions which helped them to understand inverses and compositions of functions better. With regard to the teaching method used, the experimental group felt that the PBL approach really enhanced their understanding whereas the comparison group felt that the lecture method detached them from the teacher since the teacher has to preach to them while they were listening (see E7, E8 and E10). The experimental group further stated that the employed method helped them to see that there are many ways to approach a mathematical problem when they are learning in groups. With regard to the type of questions which were asked both groups acknowledged that they were very relevant and enhanced their understanding of inverses and compositions of functions (see E3, E2 and E5).

On what the students considered to be the most important aspect in the teaching and learning of inverses and compositions of functions the experimental group felt that reading to understand the question is the most important aspect (see E1), whereas the comparison group felt that one should be good in understanding the mathematics in English language (see C1). On the aspect of how the students got to know that they had mastered inverses and compositions of functions, both groups acknowledge that they have to do several mathematical problems and compare their solutions with their group members or with people they felt are better than them. Both groups stated that they would see if their answers make sense (see C8, C7, E6 and E10). Carlson et al. (1999) state that one's ability to self-reflect on his solution; checking of the solution and justifying every process is an important aspect that will help them to seek remedy from those they think might be superior to them in the learning process.

On what the students felt were the qualities of a good tutorial session both groups felt that a good tutorial session should involve both the lecturer and the students working together in solving problems (see E9 and C1). The experimental groups felt that in a tutorial session the teacher should also consider what comes from the students and should not be teacher dominated, but the comparison group felt that a good tutorial session should start with the students alone then the group and then the teacher is involved finally in a group discussion then a class discussion (see E2) to enhance better understanding. These results corroborate with the findings of Conoldi (2008) who stated that students feel confident that they can solve problems. Both groups felt that a good tutorial session should connect what has been taught in class with what is in the tutorial session. The comparison group stated that a good tutorial session should maintain a small number of students four or five at most (see E4 and C4). Dolmans et al. (2005) state that tutorial groups in a learning environment should comprise of leaners of students between five and eight for easy coordination and maximum corporation among group members solving problems together.

On what students felt were the most challenging aspects, both groups acknowledged that formulating a composition function from a word problem was a challenging aspect. Apart from that both groups found the domain and range to be a very difficult concept especially if one is given logarithmic, exponential and trigonometric inverses and compositions of functions (C4, C7 and E8). The experimental group felt that interpreting a composition function was a very difficult (see E4). On the aspect of what their roles would be if they were put in a tutorial group both groups acknowledged the aspect of eliciting what they know so that they can be corrected by other group members. The experimental group students stated they would just listen to what they are saying, while the comparison group members stated that they would listen first to what they are saying then they will give their input (see E6).

On the roles of the various teaching approaches the experimental group stated that PBL made students think faster and trained them to understand mathematics better through the use of the real world scenarios (see E3, E2 and E5). On the other hand the comparison group students felt that the role of the lecture method was to push the syllabus, and confuse the students more and more (C3, C11 and C9). On the aspect of how frequently a particular teaching method should be used the comparison group students felt that the employment of the lecture method alone without mixing it with other methods makes the teaching and learning boring (see C6). They suggested that it should be employed with tutorials and group work to make the learning more meaningful (see C11). The experimental group stated that PBL made learning interesting and further stated that mathematics tended to be boring if it is teacher dominated but with PBL you interact with realities that will help you to think faster and be a good problem solver in future (see E5). PBL has the advantage that it provides learners having limited knowledge of a field of study with an opportunity to achieve high levels of cognitive engagement. These findings corroborate with the findings of Dolmans et al. (2005) who noted that the process of solving problems, allow learners to discover new ways to solve new real-world problems which is a tool needed by learners when they are exposed to new educational or work environments.

Regarding students' perceptions about group works, both groups agreed that they learnt better in a group through sharing what they knew (see C4 and E6). However, the comparison group stated that group learning was not beneficial if one was learning with individuals who are in competition and want to pass on their own (see C7). Concerning learning strategies the students employed which enhanced their understanding of inverses and compositions of functions both groups acknowledged their involvement in a group fostered better understanding. The members from the two groups both stated that they constantly utilized the library for them to understand better see C8, C10, E4, E3 and E5). Some members of the comparison group stated that they did not believe in group learning but doing things on their own (see C6 and C5). However, one member of the experimental group stated that she had a specific study strategy and did not understand inverses and compositions of functions at all. Even and Bruckheimer (2008) state that any teaching strategy that the teacher employs may not benefit all students in the learning environment and as such, it remains the responsibility of the teacher to make an informed follow up on the effectiveness of the employed teaching strategy so that he can use a suitable strategy that will benefit all the students. In almost every classroom there will be students who require special consideration. Remember, the teacher is not expected to provide totally different programs to students; rather, the difference be managed using a variety of approaches. The teacher should determine the best methods to address students' needs. In a typical classroom, there may be several students who require differentiated instruction (Note that all students benefit from differentiated instruction, but for some students it is essential) some students may require adaptations to instruction and assessment some students may require additional supports. Therefore, the employment of PBL could have been supported by the use of other methods in order to produce best results in the classroom.

The comparison group recommended that the lecture method should be mixed with other teaching methods to make the learning more meaningful and more student-centred than lecturer/teacher centred (see C5, C6 and C9). They advised that more real world problems should be included in teaching and learning when this method is used. The experimental group stated that the PBL approach was needed at FP and had a role in every subject. However, they advised that this approach must be implemented at the beginning of the year through out all the other mathematics topics and not only in inverses and compositions of functions (see E9, E1 and E4). They argued that spreading PBL to other FP subjects would make the teaching and learning at FP more meaningful and exciting (see E8). They further stated that PBL left them with knowledge which they are sure will be retained forever (see E4). Biggs (2009) found out that in an academic setting students need to use and describe the process of mental activity. Students who are taught using the PBL approach will have a clear understanding of ordinary thinking and awareness and understanding of thinking. Downing (2001) states that the extent of success of any learning process lies upon the provision of quality learning experiences provided to the students, of which PBL is one of those learning appoaches which can meet this goal of enhancing quality learning platforms.

With regard to the advice the students in the two groups would give to anyone intending to join the FP both groups stated that they would advise prospective students never to miss lectures and that they should work very hard from the first to the last day. Both group members also stated that they would ask the teacher they did not understand well during a lecture he/she should make an effort to find out more on that aspect before the next lesson (see C2, C4, E9 and E12).

5.14 ANALYSIS OF THE RESEARCH PROJECT RESULTS FOR THE COMPARISON AND EXPERIMENTAL GROUP

The two groups had to come up with a well-designed group project based on their understanding of inverses and compositions of inverse and /or compositions functions (see Appendix G), and was marked out of 100. The purpose of the project was to find out the students ‘experiences with the PBL approach in the teaching of composition and inverses of functions as compared to those who are taught using the lecture method, and to find out how students had understood the teaching and learning of inverses and compositions of functions by applying their gained knowledge through designing a meaningful project based on inverses and compositions of functions. Students were assessed based on the pre-disclosed assessment rubric Appendix F. The findings from the research project were meant to triangulate the findings from questionnaires, pre-post-test and the observation protocols.

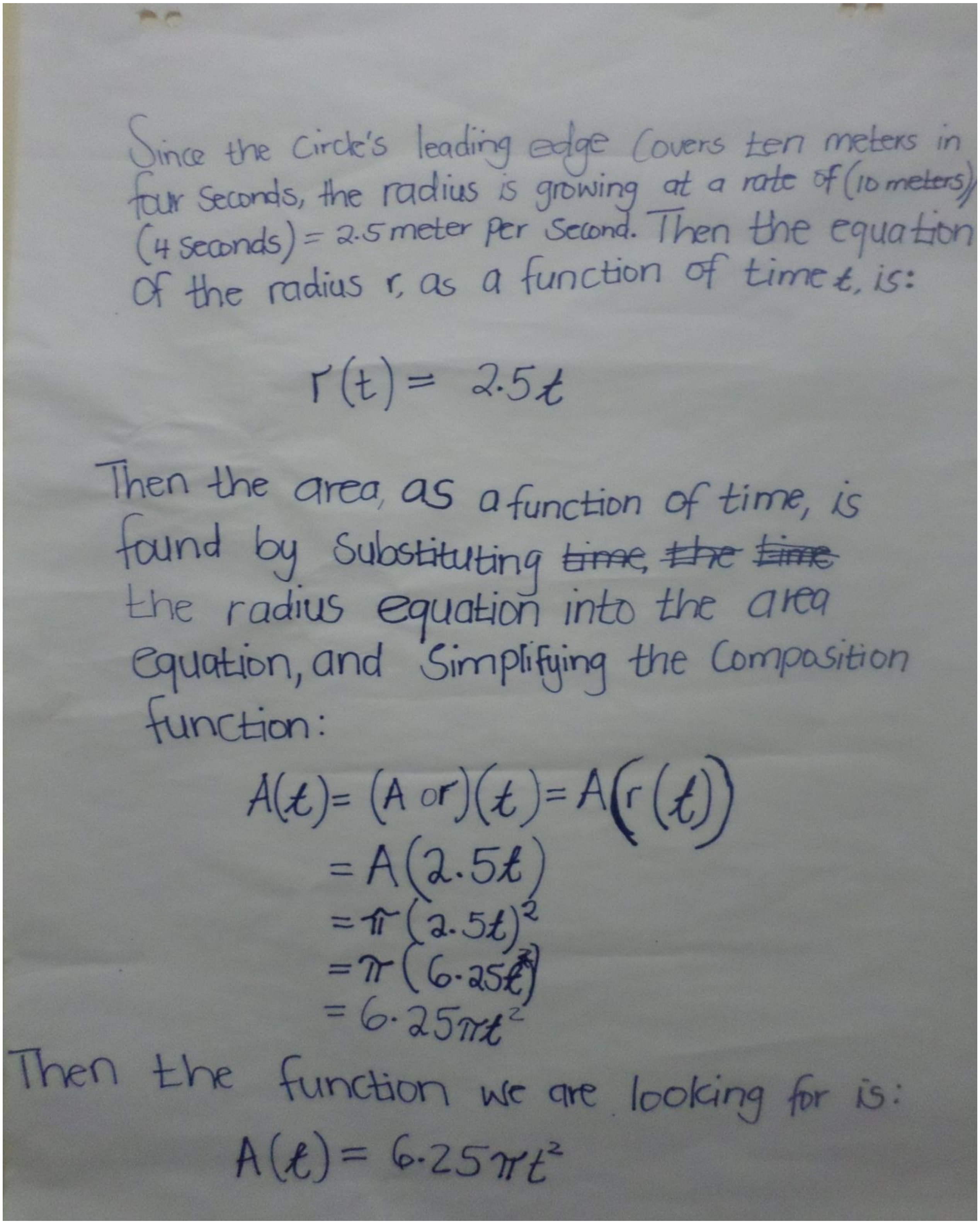
The main areas of focus as highlighted on the rubric were (1) *the process* i.e. sources of inspiration of the problem, the relevance of the problem and evidence of data collection (2) *The product* i.e. evidence that the group had made thorough use of their acquired knowledge of inverses and compositions of functions, levels of students’ creativity, evidence of collected data and then finally (3) *Oral presentation* i.e. students had to present their designed project to the class and would answer questions from other comparison group members about their designed project i.e. organization of the group presentation, evidence of group ideas well communicated and utilization of Information Technology (IT) or Visual aids in the presentation and finally the groups’ ability to answer questions coming from the audience based on their designed project. Below is an analysis of the few projects which were presented by the different groups.

5.14.1. Analysis of the research project results for the comparison and experimental group

*Comparison Group CF206 -**Topic: The applications of compositions of functions in the computer manufacturing industry***

In this project the group was looking at various applications of compositions of functions in real life various applications. Of particular importance was the following demonstration of the applications of compositions of functions in the real world where the group explored the following problem and demonstrated to the class how the compositions of functions are involved. After discussing the various applications of compositions of functions, the group took time to present the following problem:

Your computer's screen saver is an expanding circle. The circle starts as a dot in the middle of the screen and expands outward, changing colours as it grows. With a twenty-one inch screen, you have a viewing area with a 10-inch radius (measured from the centre diagonally down to a corner). The circle reaches the corners in four seconds. Express the area of the circle (discounting the area cut off by the edges of the viewing area) as a function of time t in seconds.



Since the Circle's leading edge Covers ten meters in four Seconds, the radius is growing at a rate of (10 meters)/(4 seconds) = 2.5 meter per Second. Then the equation Of the radius r, as a function of time t, is:

$$r(t) = 2.5t$$

Then the area, as a function of time, is found by Substituting ~~time, the time~~ the radius equation into the area equation, and Simplifying the Composition function:

$$A(t) = (A \circ r)(t) = A(r(t))$$
$$= A(2.5t)$$
$$= \pi(2.5t)^2$$
$$= \pi(6.25t^2)$$
$$= 6.25\pi t^2$$

Then the function we are looking for is:

$$A(t) = 6.25\pi t^2$$

Figure 5. 40: Comparison group project presentation

The Process

-The students started off by discussing the various applications of inverses and compositions of functions in real life. In their motivation to the aspect they stated was that the zeal to get to know more about compositions of functions motivated them to look into their topic. They also stated that it was out of curiosity to learn more about compositions of functions that they wanted to investigate on this matter. The group showed higher level creativity because their topic was researchable and real.

-There was a variety of ideas which were generated by the group members about the applications compositions of functions in real life and convincing reasons for deciding on the problem were well explained showing their ability to analyse.

-The students had well-defined tasks in their search for real-world applications of compositions of functions and there was logical thinking and meaningful follow up questions from the group members which showed their ability to understand.

The Product

The group made thorough use of their knowledge from the content of the theme compositions of functions which showed their understanding of the theme.

-There was vast evidence of the group having gone through the process of data collections and performing some practical experiments on their project trying what could work and what could not work until they settled at their solution.

-Group findings which were available were well analysed and the reported findings were very clear in most cases which showed their groups inability to evaluate their findings.

-Steps which were taken to solve one of the problems were very clear and comprehensive which showed some understanding of some their project.

-There was evidence that some aspects of creativity and ability to apply their new knowledge on the real applications of compositions of functions.

Oral presentation

-The group presentation was well organized and there was evidence that the group members had worked together to come up with their project.

-There was effective utilization of visual aids in the presentation which showed creativity.

-Group ideas and findings were presented and well understood in most cases.

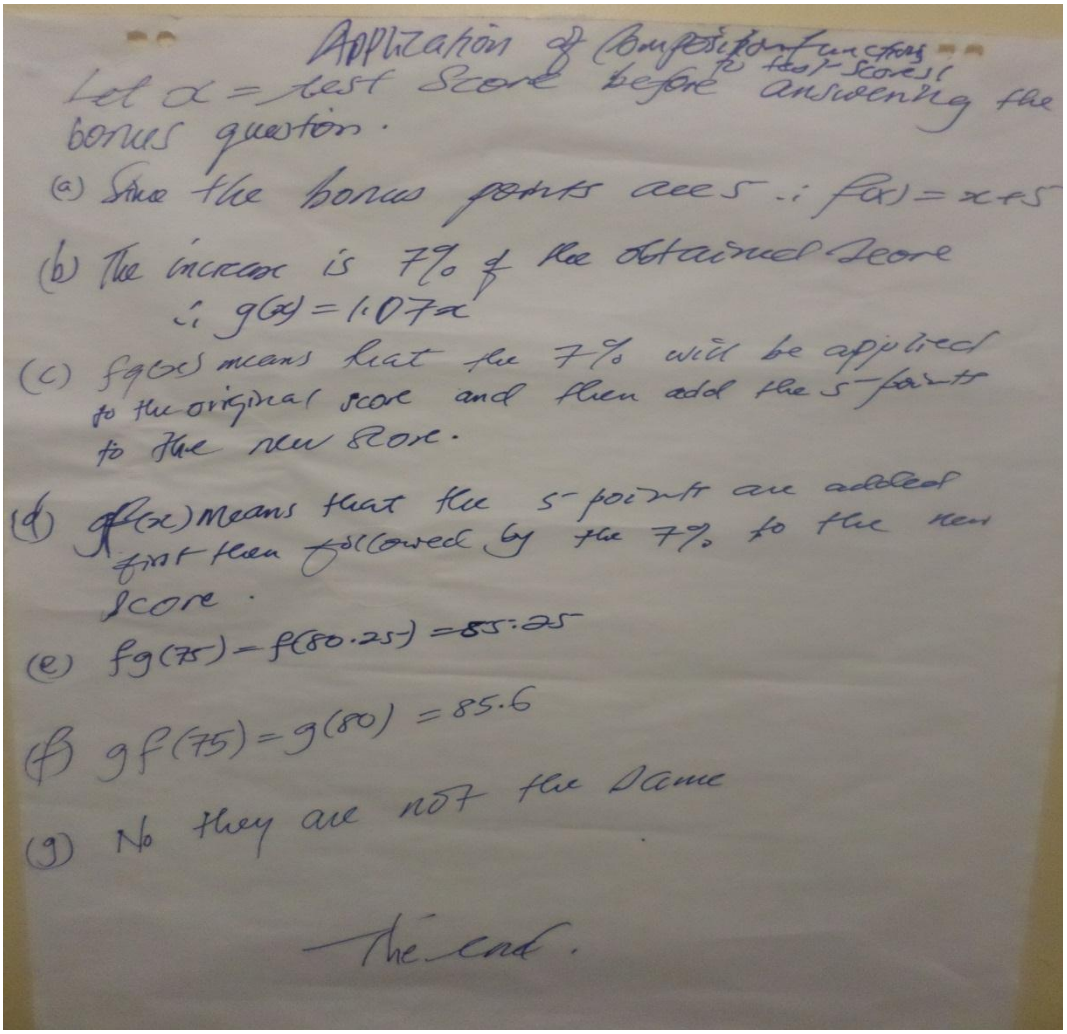
-The group managed to answer questions from the audience about the presentation they made about their project, which to a greater extent showed understanding of the group project.

*Group CB202**: Topic: Application of composition functions in calculating test scores.***

A mathematics test has a bonus question. The directions simply state that if you answer the question correctly you will be awarded 5 bonus points and your test grade will be increased by 7% of your score.

Let x = test score before answering the bonus question.

a. *Write a function, $f(x)$, to represent just the 5 bonus points.*
b. *Write a function, $g(x)$, to represent just the percent of increase.*
c. *Find $fg(75)$*
d. *Explain the meaning of $fg(x)$*
e. *Explain the meaning of $gf(x)$*
f. *Find $gf(75)$*
g. *Does $fg(x) = gf(x)$ in this problem?*



Application of Composition functions to test scores

Let x = test score before answering the bonus question.

(a) Since the bonus points are 5 $\therefore f(x) = x+5$

(b) The increase is 7% of the obtained score $\therefore g(x) = 1.07x$

(c) $fg(x)$ means that the 7% will be applied to the original score and then add the 5 points to the new score.

(d) $gf(x)$ means that the 5 points are added first then followed by the 7% to the new score.

(e) $fg(75) = f(80.25) = 85.25$

(f) $gf(75) = g(80) = 85.6$

(g) No they are not the same

The end.

Figure 5.41: Comparison group project presentation

Process

- The group stated that they were inspired by the way the teacher was connecting functions to the real world and felt that they could also go an extra mile to discover something on their own.

-This problem was real and was very relevant to the topic compositions of functions since it was looking at how marks and bonus marks were put together in a meaningful mathematical argument for compositions of functions, this showed higher level of creativity.

-There was a statement of the problem of the given by the group which was stated as “How can composition functions be used in calculating test scores”?

-The group generated a variety of ideas about the problem and there were convincing reasons for deciding on the problem which showed the groups' ability to analyse their problem.

-The students in this group had a well-defined task and showed good and logical thinking skills in their search for solutions to the problem, meaningful follow-up questions on the problem were asked, which showed the students' higher levels of understanding.

The Product

-The group made thorough use of their knowledge from the content of the theme compositions of functions which showed their understanding of the theme.

-There was a lot of evidence that the group carried out some experiments and data collection on the matter.

-The group findings were well analysed with different trials being drawn on the groups' worksheets and were well filed in the groups' file for data collection.

Oral presentation

-The group presentation was well organized and there was evidence that the group members had worked together to come up with their project since each member of the group could answer questions based on the group presentation.

-There was effective utilization of visual aids in the presentation which showed creativity

-Group ideas and findings were well presented and clearly understood.

-The group successfully answered all questions from the audience about the presentation they made about their project which showed great understanding.

5.14.2. Analysis of the research project presentation results for the Experimental group

One of the requirements of PBL is that ability of students to design a project based on whatever they might have been focusing on (Barrows, 1996, 2002). The experimental group had to come up with a well-designed group project based on their understanding of inverses and/or compositions of functions as was done by the experimental group Appendix G, which was marked out of 100. The purpose of the project was to assess how students had understood the teaching and learning of inverses and compositions of functions by applying their knowledge to design a meaningful project. For the comparison group, students were assessed based on the pre-disclosed assessment rubric in Appendix F.

The main areas of focus as highlighted on the rubric were (1) *the process* (2) *The product* inverses and compositions of functions (3) and *Oral presentation* as explained for the comparison group above.

Another fascinating group project was made by group PJ112, the outline of the project is shown in the summary below.

*Group EA112: -**Topic: The applications of compositions of functions in the retailing business***

- You make a purchase at a local hardware store, but what you've bought is too big to take home in your car. For a small fee, you arrange to have the hardware store deliver your purchase for you. You pay for your purchase, plus the sales taxes, plus the fee. The taxes are 7.5% and the fee is $20.

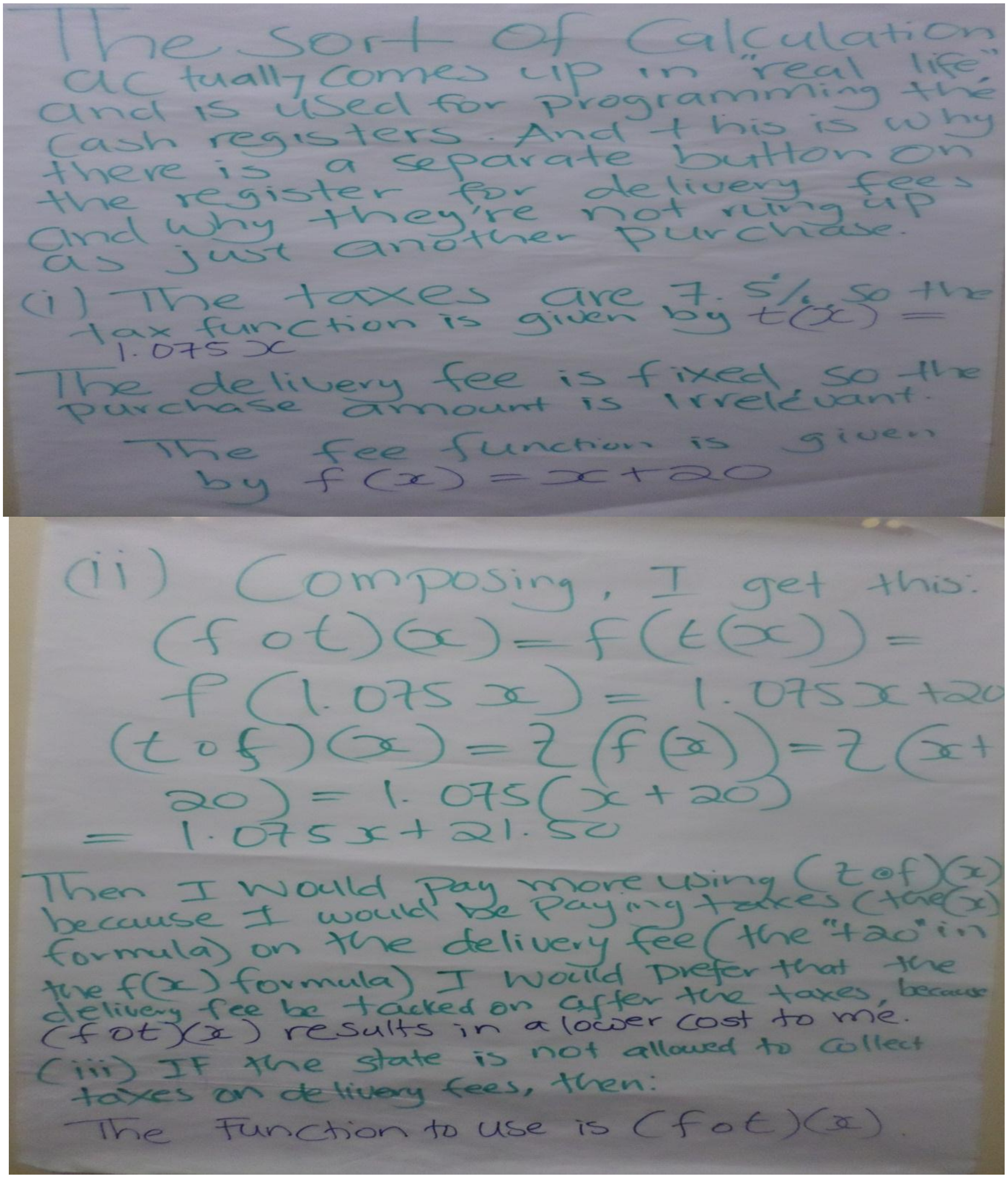
- *i. Write a function t(x) for the total, after taxes, on the purchase amount x.*
- *ii. Write another function f(x) for the total, including the delivery fee, on the purchase amount x.*
- *iii. Calculate and interpret (f ₒ t) (x) and (t ₒ f) (x). Which results in a lower cost to you?*
- *iv. Suppose taxes, by law, are not to be charged on delivery fees. Which composite function must then be used?*

-The group stated that they were inspired so much by the way the teacher was connecting compositions of functions to the real world and wanted to apply their accrued knowledge to solve real and practical problems.

-This problem was real and was very relevant to the topic compositions of functions since it was looking at how compositions of functions can be applied to the retail business and this showed higher levels of creativity.

-There was no outlined statement of the problem of the given by the group though the group stated in short what their intention was.

-The group generated a variety of ideas about the problem to be investigated and there were convincing reasons for deciding on the problem which showed the groups' ability to analyse.


The sort of calculation actually comes up in "real life" and is used for programming the cash registers. And this is why there is a separate button on the register for delivery fees and why they're not rung up as just another purchase.

(i) The taxes are 7.5% so the tax function is given by $t(x) = 1.075x$

The delivery fee is fixed, so the purchase amount is irrelevant.

The fee function is given by $f(x) = x + 20$

(ii) Composing, I get this:

$(f \circ t)(x) = f(t(x)) = f(1.075x) = 1.075x + 20$

$(t \circ f)(x) = t(f(x)) = t(x + 20) = 1.075(x + 20)$
$= 1.075x + 21.50$

Then I would pay more using $(t \circ f)(x)$ because I would be paying taxes (the $t(x)$ formula) on the delivery fee (the "+20" in the $f(x)$ formula) I would prefer that the delivery fee be tacked on after the taxes, because $(f \circ t)(x)$ results in a lower cost to me.

(iii) If the state is not allowed to collect taxes on delivery fees, then:

The function to use is $(f \circ t)(x)$.

Figure 5.42: Experimental group project presentation

--The students in this group had a well-defined task and showed good and logical thinking skills in their search for solutions to the problem, meaningful follow-up

questions on the problem were asked, which showed the students' understanding of the problem.

The Product

-There was thorough use of their knowledge from the content of the theme compositions of functions which showed their understanding of the theme.

-There was tangible evidence that the group carried out some experiments and data collection on the matter for quite a long time, trying what could work and what could not work.

-The group tried many alternatives in coming up with the correct answers in some cases trial and error was employed and some solutions were disregarded in the process with clear justification of their rejections, this was an art of creativity on the part of the students.

Oral presentation

-The group presentation was well organized and there was evidence that the group members had worked together to come up with their project since each member of the group managed to say something during the course of their presentation.

-There was effective utilization of visual aids in the presentation which showed creativity by the group members involved.

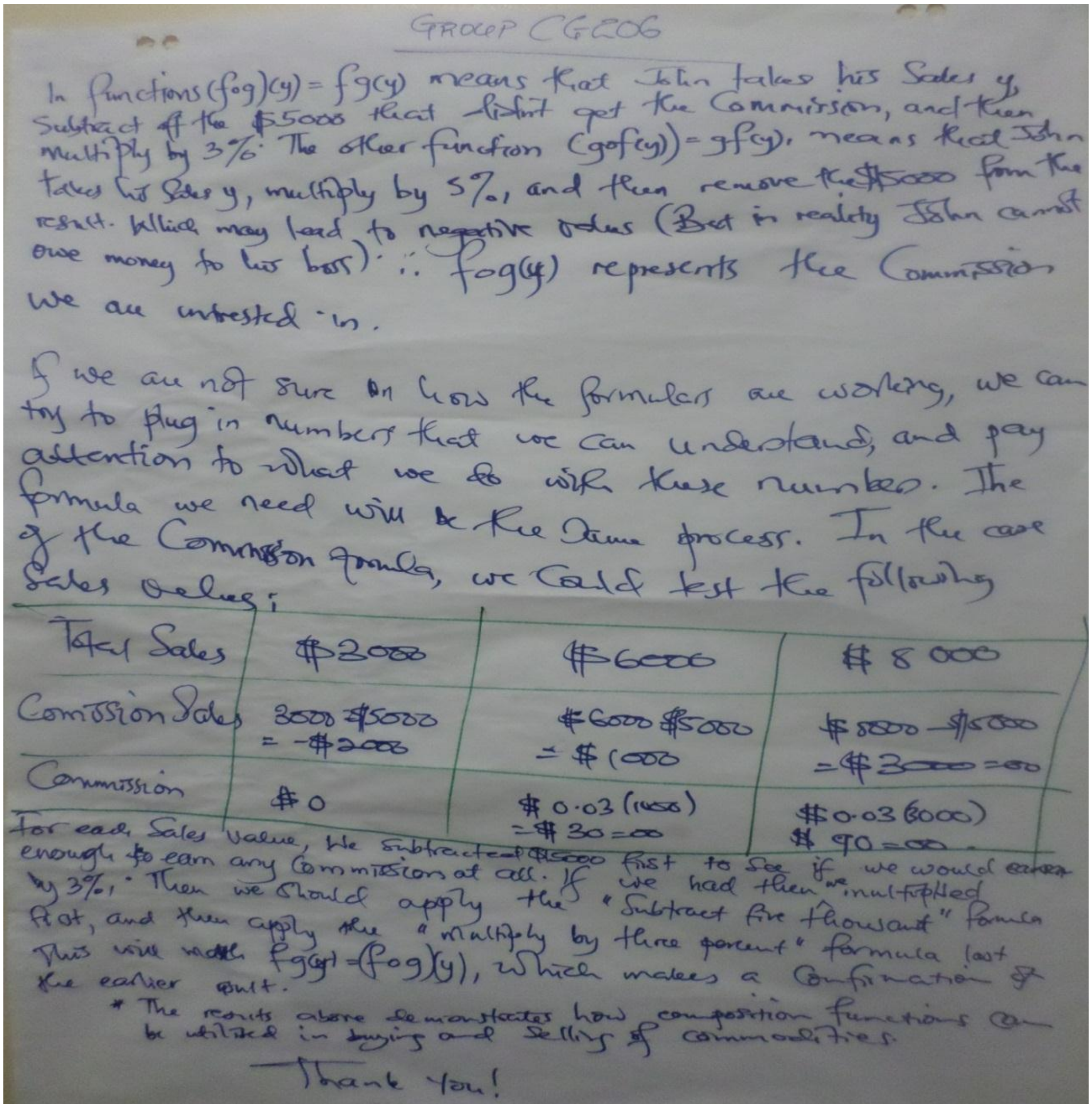
-Group ideas and findings were well presented and clearly understood.

-The group members managed to successfully answer all questions from the audience about the presentation they made about their project which showed great understanding from the group members.

*Group ED102**:-Topic: Application of composition functions sales.***

The group started off by discussing various applications of composition functions before they presented the problem below which became their project focus.

John works 40 hours a week at a furniture store. He receives a $220 weekly salary plus a 3% commission on sales over $5000.Assuming that he sells enough this week to get the commission. Given the functions g(y) =y-5000 and f(y) =0.03y, which of ($g_o f$) (y) and ($f_o g$) (y) represents his commission.



GROUP CG206

In functions $(f\circ g)(y) = fg(y)$ means that John takes his Sales y, Subtract off the \$5000 that didn't get the Commission, and then multiply by 3%. The other function $(g\circ f(y)) = gf(y)$, means that John takes his Sales y, multiply by 5%, and then remove the \$5000 from the result. Which may lead to negative values (But in reality John cannot owe money to his boss). ∴ $f\circ g(y)$ represents the Commission we are interested in.

If we are not sure on how the formulas are working, we can try to plug in numbers that we can understand, and pay attention to what we do with these numbers. The formula we need will be the same process. In the case of the Commission formula, we could test the following Sales values;

Total Sales	\$2000	\$6000	\$8000
Commission Sales	3000 \$5000 = -\$2000	\$6000 \$5000 = \$1000	\$8000 - \$5000 = \$3000=00
Commission	\$0	\$0.03 (1000) = \$30=00	\$0.03 (3000) \$90=00.

For each Sales value, we subtracted \$5000 first to see if we would earn enough to earn any Commission at all. If we had then we multiplied by 3%. Then we should apply the "Subtract five thousand" formula that, and then apply the "multiply by three percent" formula last. This will make $fg(y) = (f\circ g)(y)$, which makes a Confirmation of the earlier result.

* The results above demonstrates how composition functions can be utilized in buying and selling of commodities.

Thank You!

Figure 5.43: Experimental group project presentation

The Process

-Various applications of inverses and compositions of functions in real life were explored by the group in their introduction of the presentation. In their motivation to the aspect they stated was that they found out that their fellow students did not understand inverses and compositions of functions well that is why they felt they would do a research on the applications of composition functions in real life. They also stated that it was out of curiosity to learn more about compositions of functions that they wanted to

investigate on this matter. The group showed higher level creativity because their topic was researchable and connected well to the real world.

-There was a variety of ideas which were generated by the group members about the applications of compositions of functions in real life and convincing reasons for deciding on the problem were well explained showing their ability to analyse.

-The students had well defined tasks in their search for real-world applications of compositions of functions and there was logical thinking and meaningful follow up questions which showed their ability to understand.

The Product

-The group made thorough use of their knowledge from the content of the theme compositions of functions which showed their understanding of the theme.

-There was vast evidence of the group having gone through the process of data collections and performing some practical experiments on their project.

-The group findings which were available were well analysed and the reported findings were very clear in most cases which showed their groups inability to evaluate their findings.

-Steps which were taken to solve one of the problems which were presented were very clear and comprehensive which showed some understanding of some their project.

-There was evidence that some aspects of creativity and ability to apply their new knowledge on the real applications of compositions of functions.

Oral presentation

-The group presentation was well organized and there was evidence that the group members had worked together to come up with their project.

-There was effective utilization of visual aids in the presentation which showed creativity.

-Group ideas and findings were presented and well understood in most cases.

-The group managed to answer questions from the audience about the presentation they made about their project, which to a greater extent showed understanding.

5.14.3. Comparative analysis of the assessment of experimental and comparison group projects

The project which the students were working on focused on teaching important knowledge and skills derived from classroom standards and key concepts at the heart of the mathematics concept- compositions and inverses of functions. Students in both groups were provided with adequate opportunities to build 21st century competencies and were provided with the guidelines and expectations together with the assessment rubric prior to doing the project for them to know the main focus areas of the teacher. The main purpose was to get students involved in the project as much as they could. Students worked in collaborative teams that employed the ideas of all group members when completing the project task. The students in both groups used series of peer critique cycles which was done by presenting their projects in the presence of their peers in their first round review. The researcher used an "anti-model" that showed what to do and what not to do.

Some of the projects which were presented by the comparison group had no titles of what the students were trying to find out and as such they ended up losing marks for that. Students from both groups managed to motivate their sources of inspiration for the problem. However, the comparison group members lacked collaborative engagement to a greater extent. Apart from that, some of the comparison group projects were not related to the theme inverses and/or composition of functions. Most of the experimental group members made clear explanations of what their projects were about, and their objectives about the problem were also very clear in most cases. Their problems were realistic and very clear and were connected to real life events though all the presented projects were on compositions of functions and none of them focused on inverses of functions. Most experimental group members had already adopted the spirit of self-regulated learning so they did not bother the teacher much on directions on how to embark on the project as was done by the comparison group members who would get to a deadlock and would not know what to do. The idea of learning together and from each other was respected by the experimental group but the comparison groups were not working together and during some of the group presentations some comparison group members would argue among themselves because it was one person's idea and not a group idea and they would beg to differ in the eyes of the public. Pawson et al.(2006) state that when embarking on a project as a group each member of group must be involved fully and should show their total commitment. Pawson et al. further state that if group members have to argue during the presentation of a project this is a clear sign that they were neot working together in the first place.Such lack of group cooperation has always failed group project work. The students were required to investigate and write a project on one aspect of inverses and compositions of functions but most of the comparison group members were investigating many things and their investigations were not focused as was expected. One aspect which the researcher was focusing on during the presentation of the projects by students was the existence of research question which was open-ended, and allowed students to develop more than one reasonable complex solution.

Azer (2000) states that through the use of research projects, students are able to apply their acquired knowledge into practice and this will improve their understanding of the learned idea as they bring together their theoretically constructed knowledge into practice. The research question had to be understandable and clear to the class. Barret (2004) states that lack of clear research questions and objectives in many cases has failed the implementation of project based learning. Based on this research question that they had developed, it was expected that students would gain the intended knowledge, skills and understanding of inverses and compositions of function. Most of the comparison group members did not have a proper research question; they either had a simple answer or were flawed as compared to most of the projects which were presented by the experimental group members.

A final retrospective analysis of the students' PBL experiences and level of understanding in the teaching of compositions and inverse of functions was done in the form of a post-test which was administered at the end of the teaching episode. The main obstacle which was encountered by the students at the PBL implementation level where they had to change their roles to problem solvers who were just given a glimpse of insight on the problem by the lecturer who took the role of a facilitator. At the beginning students had a problem with the interpretation of the long and winding word problem into a valid mathematical expression which they would work with later. As time unfolded, students got used to working with these types of problems as they researched more in the library and also utilizing the internet. Questions which were pursed during group presentations created a constructive academic friction among the learners as they gave alternatives to their solutions. In these PBL sessions, students were exploring new innovative ways of solving problems, a scenario that made the researcher also learn new ways of handling and solving the PBL questions. This also made the researcher to self- reflect on his own frame of reference, and justify his rejections or acceptance of the presented group findings. Experimental group performed exceptionally well in their presented projects and also articulated well in the presentation of the projects. They showed a better understanding of project based learning than their comparison group counterparts. This was really a successful endeavour.

5.15. ANALYSIS OF THE RESEARCH PROJECT QUANTITATIVE RESULTS FOR THE COMPARISON AND EXPERIMENTAL GROUP

The table 5.15 below shows the project quantitative results according to the HLT domains which were used on the assessment rubric.

Table 5.15: Experimental group Projects group assessment results by rubric domains

Domain	Hypothetical Learning Trajectory Domain	Domain Total Marks	% weight	total marks obtained Experimental GP students	Total possible marks by 40 students	Domain Average percentage	Variance
D1	Process	25	25.0	160.26	200	80.13	8.61
D2	Product	45	45.0	246.47	360	68.6	15.49
D3	Oral presentation	30	30.0	174.1	240	72.54	10.32

Table 5.15 shows that the experimental group got a group average of 80.13% in the process domain, while they got had 68.8% in the product domain and finally they had a group average mark of 72.54% on their oral presentations.

The control group also had research project on inverses and compositions of functions which they presented to the class. Table 5.16 shows the control group results for the process, product and oral presentation domains which were used in the study.

Table 5.16: Control group project group attainment results by rubric domains (N=12)

Domain	Hypothetical Learning Trajectory Domain	Domain Total Marks	% weight	total marks obtained Control GP students	Total possible marks by 8 groups	Domain Average percentage	Variance
D1	Process	25	25.0	103.12	200	51.56	6.4
D2	Product	45	45.0	241.62	360	66.94	11.6
D3	Oral presentation	30	30.0	147.74	240	59.47	7.73

Table 5. 16 show that the control group has a domain total of 51.56% on the project process, 66.94% on the product and 59.47% on their oral presentations.

The quantitative results for the experimental and comparison group for the projects were awarded after the presentation phase of the project. A total of twelve projects were collected for each group and were marked out of 100. Table 5.17 below shows a summary of the results of the results of the two groups.

Table 5. 17: Quantitative results for the projects for both groups (N=12)

Group	Range	Minimum	Maximum	Mean Score out of 100		Std. Deviation	Variance
					Std. Error		
Comparison	26	50	76	61.56	0.949	5.076	25.765
Experimental	32	57	89	72.63	2.312	5.868	34.432

The table shows that the comparison group had a mean of 61.56 with a standard deviation of 5.076 with a maximum value of 72 and a minimum value of 50 with a range mark of 26, whereas the experimental group had a mean project mark of 72.63 with a standard deviation of 5.868 with a maximum project mark of 89 and a minimum mark of 57 with a range of 32.

At a glance these results show that the mean of the experimental group is higher than that of the comparison group. A two tailed *z* hypothesis test of the quantitative projects results of the two groups showed that ($Z_{Calculated}$=4.9425, $Z_{Standard}$=1.96; α=0.05) and rejected the null hypothesis which stated that (H_0:$\mu_E - \mu_C = 0$) in favour of the alternative which stated that (H_1:$\mu_E - \mu_C \neq 0$).These results show that the experimental group performed significantly better than the comparison group on the overall project result analysis. These results concur with the findings by Nguyen (2009) who applied PBL approach on medical students and noted significantly better performance ability among the PBL group which used real-life problems that cross traditional boundaries than to those students who were taught using the conventional approach. Nguyen (2009) further argue that though the conventional teaching approaches that have been in use for decades in higher education have been found to limit students in becoming proper problem solvers and self-directed learners in the information age.

5.15.1 Comparative analysis of the experimental and comparison group results by HLT domains

The project results for the experimental and the control group were analysed by HLT domains on the rubric. Table 5.18 shows the results of the carried out hypothesis on the domains.

Table 5.18: Experimental group's project assessment results analysis by HLT domain (N=12)

Domain	Hypothetical Learning Trajectory Domain	Domain Total Marks	% weight	% mean mark for Experimental group	Variance(Pre)	% mean mark for comparison group	Variances (C)	z-test result for independent groups	Critical value of *t* for α=0.05 and	Decision on null hypothesis
D1	Process	30	30	80.13	8.61	51.56	6.4	25.5452	1.96	Reject H_0
D2	Product	45	45	68.6	15.49	66.94	11.6	1.105	1.96	Accept H_0
D3	Oral presentation	25	25	72.54	10.32	59.47	7.73	10.382	1.96	Reject H_0
	Total	**100**	**100**							

The project results in table 5.18 reflect that on the Process and Oral presentation domains the experimental group performed better than the comparison group (see decision rule above).However, on the final Product there was no significant difference between the experimental group and the comparison group. These results suggest that the exposure of the experimental group to intensive academic cohesion as they interact with each other could have helped them to perform better the comparison group in their oral presentation skills. On the Process domain the experimental group's involvement in the search for answers during the PBL classes using the hybrid PBL model (see 2.8.9) could be the justification for the higher performance.

5.16. CONCLUSION

This chapter presented the findings and their interpretations. The results of the study reflected that at pre-test the performances of the two groups (comparison and experimental) were the same. But after the implementation of the treatment which was the PBL to the experimental group the experimental group performed significantly better than the comparison group in the overall quantitative data analysis for the pre-post-test and the group projects. However, an individual domain performance analysis for the pre-post-test shows that the experimental group performances were only significant in one domain D2 but in all the other domains did not show a significant different in performance between the two groups as expected. This study also found out that PBL had the potential to improve students' performance based on the results from focus group interview and the self-administered questionnaire. Analysis of the overall analysis of the quantitative results of the presented projects also showed that the students from the experimental group performed significantly better than the students from the comparison group which was a further confirmation of the effectiveness of the PBL approach to teaching. However, in terms of individual domain comparison there was only a statistical difference on the process and the oral presentation domains, but on the product there was no significant difference in the performances of the two groups.

It was also revealed in this study that the employment of PBL should be coupled with other methods in order to make it more beneficial to the students. Apart from that, this study also found out that though the conventional lecture method did not produce results which were comparable to the PBL approach overall, but its role in the teaching and learning of compositions and inverses of functions is very significant in improving the understanding of students. This study found out that the use of the lecture approach should be coupled with other more innovative and student friendly teaching approaches for it to improve foundations students' performance in the teaching and learning of inverses and compositions of functions. In this study the dilution of the lecture method through the use of tutorials and group work could have been the factor that helped to make performances comparable to the PBL group.

Chapter 6 will discuss the findings in connection with the purpose of the study and will also present the theoretical gap which this study contributed towards, then the areas for further research together with the conclusion and recommendations of the study.

CHAPTER SIX

DISCUSSIONS, RECOMMENDATIONS AND CONCLUSION OF THE STUDY

6.1 INTRODUCTION

This chapter (chapter 6) has the following sections: in the first section the researcher presents a summary of the results, followed by a discussion of the results, then the research gap which this study intended to cover, and finally recommendations of the study.

6.2. SUMMARY OF THE STUDY CHAPTERS

Chapter 1 gave an introduction and the background of the study, the rationale and significance of the study. It then gave an overview of the Namibian education system before and after independence. Philosophical principles underpinning FP and their role in higher education were also discussed. Apart from that, it gave a brief overview of PBL and its origin followed by the statement of the problem. It then presented the questions of the study, assumptions, limitations and delimitations of the study.

Chapter 2 gave a review of literature on teaching and learning at university level. It then gave detailed explanations of the various approaches to PBL, the various features of PBL, its advantages and disadvantages. In addition to that, it addressed the general implementation, assessment and organization processes of PBL. The models and literature were used to clarify the research focus. This chapter made a clarification on the link between realistic mathematics and PBL. Finally the chapter then presented the hypothetical learning trajectory definitions suitable for FP.

Chapter 3 of the study reviewed literature on functions, their evolution, relevance to human life, and their applications to mathematics. It also explored the different types of functions including (composition and inverses) and the various frameworks used in the understanding of functions. Finally, literature was also used to clarify the research focus.

Chapter 4 gave the hypothetical methodological approaches used in this study. Research setting and timing was also explained. This chapter also presented research ethics, instrumentation sample and sampling procedures and clarification of data analysis. This chapter also presented an alignment of the research questions to the instruments and the statistical tests which were used to confirm the study findings. Finally, it gave the PBL implementation phases which were used in the study.

Chapter 5 presented the data from the various data collection procedures of the research study emanating from the comparison and experimental groups of the study during implementation of the PBL approach in the teaching of compositions and inverses of functions.

Chapter 6 presented a summary of the main findings of the study and showed how they were aligned to the main aims, objectives and research questions, the research gap which this study fulfilled and the recommendations as well as areas for further research.

6.3 SUMMARY OF MAIN FINDINGS

The main focus of the study was on the understanding of compositions and inverses of functions taught using the PBL approach while the general concept of a function played a secondary role. The main objective was to determine the effect of the PBL approach on the mathematics performance of the science foundation students of the University of Namibia in the teaching and learning of compositions and inverses of functions. The two groups of students (PBL and Comparison (Lecture)) were taught using two approaches. At the pre-test level, students from both groups could not precisely define and represent a function graphically but after being taught, students from both groups could define a function. The students from both groups could also relate everyday events to the function concept. It was envisaged that the experimental group would perform better in this domain but the results of the *z* carried out showed no significant difference in the way the students from the two groups understood the definitions of a function. The comparison group members could also relate functions to everyday events as the experimental group members as reflected by the z test carried out on this domain. The focus of the study was to find out how students' performances would improve as a result of their PBL experiences. The students who were taught using the PBL method could both define and relate the composition and inverse functions to real world events. One of the roles of PBL is to engage students in selfless sharing of academic information, with the purpose of creating independent and autonomous thinkers who have the ability to solve dynamic real life problems, whereas the lecture approach allows more students to be taught in one classroom environment.

As stated in chapters 4 and 5, the focus group and questionnaire questions that focused on definitions of the function were meant to open up the discussion and allow students to have a better understanding of what functions are before coming to compositions and inverses of functions, which are concepts embedded in functions. In the section that follows, the researcher presents a summary of the findings grouped according to the following conceptual domains which were used for data analysis for the study.

(1) Students' perceptions with regard to a particular teaching approach - B1: For effective teaching to transpire in a classroom there must be concerted efforts from both the teacher and the students. One of the research questions of the study was to find out what the student perceptions were with regard to the teaching method which was employed in the teaching and learning of compositions and inverses of functions. The experimental group showed positive perceptions during the teaching and learning of compositions and inverses of functions and felt that composition functions were relevant to their lives since they connect well with other mathematical topics, hence the reason for learning about them (see 5.12.2-E4). They stated that group interactions improved their ability to communicate and share ideas with other students during the teaching and learning of compositions and inverses of functions (see 5.12.2-E6; E7; E3).

The comparison group students found the lecture method coupled with the use of tutorials and group work to be helpful and their perceptions with regard to this teaching approach was positive since whatever was done during the lesson was meant to benefit them. In addition to that, they found the teaching and learning of compositions and inverses of functions to be very relevant to them because functions are connected to other mathematics topics like subject of formula, equations and graphs, which form part of the foundation mathematics curriculum. They were able to come up with group projects which they presented to the class and this greatly enhanced their understanding. However, some comparison group members felt that the type of problems which they were doing in class were far detached from what they were seeing in past exam papers and tests for bridging course for undergraduate mathematics (see 5.12.2-C7); hence they felt that probably the teacher was going out of syllabus.

(2) Definition of a function-B2: the focus of the study was on compositions and inverses of a function but the researcher had elicited what the students knew about a function in general. At pre-test, students from both groups could not give a proper definition of a function but they could give algebraic examples of a function (see 5.4.1.1 and 5.2.1. 1). The reason for this might be because the high school emphasis of functions as a topic is only done in higher level mathematics of which most of the FP students did either core or extended mathematics. At post-test, students from both groups could define a function and could give a relevant real life example thereof, while the comparison group members could define a function from an algebraic perspective and could also give relevant algebraic examples of a function (see 5.8.2.1 and 5.8.3.1). The difference in the two groups with regard to the definitions of a function was that the comparison group could not give a real world example of a function in their definitions. At post-test, some members of the experimental group defined a function as a set of ordered pairs which showed their extended understanding of a function based on their experiences with PBL (see 5.8.3.6). There was no significant difference in terms of students' performances in this domain for the two groups. The two methods were comparably effective.

(3) Definition of composition and inverse functions-B3: With regard to the definition of a composition function, none of the students from either group managed to give an acceptable definition of a composition function at pre-test. However, at post-test, most members of the experimental group could define and give relevant real-world example of a composition function but students some students from the comparison group could still not define the composition of a function. All they could do was to give examples of a compositions and inverses functions. On the inverse of a function, none of the students from either group could define the inverse of a function at pre-test, but students from both groups could find the algebraic representation of the inverse of a function through an algebraic formula. On this aspect students from both groups demonstrated a good procedural knowledge for the inverse of a function. The experimental group members

could define and give an acceptable real world example of the inverse of a function. However at post-test, some members of the experimental group went an extra mile to state some of the properties of composition functions e.g. the non-commutative property and some could state the domain and range of a composition and inverse of a function, though in some cases domain and range would be confused by some students. At post-test, students from both groups could relate a function to its inverse graphically and they were also aware of the fact that only a one-to-one function is invertible. There was no significant difference in performances for students in the two groups on this domain.

(4) Formulation of a composition and inverse of a function from a word problem-B3.At pre-test, none of the students from either group could formulate either a composition of a function or its inverse. However, there was an improvement in the students' understanding of the formulation of compositions and inverses of functions by members from both groups. The comparison group students were applying the formulaic rule to get their results with little understanding of the real-world significance comparison. However, there was no significant difference in the performances of the students from the two groups on this domain based on the carried out *z* test (see 5.8.1).

(5) Manipulation of abstract expressions involving compositions and inverses of functions-B5: At pre-test none of the members of either group could perform any manipulations involving compositions of functions since this was a new concept to all members. However at post-test, there was an improved understanding of the concept from both groups and students could use the four basic operations in composing and inverting functions (see 5.8.2.5 and 5.8.3.5). There was no significant difference in the performances of the students in the two groups based on the z test which was carried out (see 5.8.1).

(6) Representation of a function-D5: At pre-test, only a few students from both groups could represent a function. The most common representations used were arrow diagrams. At post-test, students from both groups could formulate a function and represent it in a variety of ways; as a set of ordered pairs, using arrow diagrams, using the Cartesian plane and using tables of values. The experimental group members could both formulate a composition function from word problems and represent it and answer

word problems involving them with little difficulty; whereas the members of the comparison group struggled to formulate composition functions from real life problems. On this domain, the experimental group participants demonstrated a better understanding of the representation of both inverses and compositions of function given only the graph of the function. This was the only domain which in which the experimental group showed a significance statistical difference in performance (see 5.8.1).

(7)The helpfulness of the tutorial sessions-B7: With regard to the helpfulness of the tutorial sessions conducted, participants from both groups confirmed that the tutorial sessions were helpful and allowed them to learn from each other as they constructed new knowledge. One member of the experimental group stated that PBL tutorials left him with knowledge that left him a changed person in terms of the way he used to perceive mathematics. However, some members of the comparison group highlighted that they did not believe in group learning but individual learning.

(8)The students' perspectives on the meaning of group work-B8: What was common to both groups on this aspect was that group work was very beneficial to their learning, and that participants learnt from each other. This greatly enhanced their understanding of compositions and inverses of functions. Some comparison group members stated that they did not benefit from the groups because they were in groups where there were selfish people who did not want to share their ideas. One member of the comparison group stated that group work allowed him to make connections of composition and inverses of functions with other mathematical topics.

(9) Sufficiency of the conditions necessary to support PBL/lecture-B9: The main purpose of the study was to find the effect of the PBL approach in the teaching and learning of compositions of functions. Therefore, the researcher had to set conditions which were necessary for PBL/lecture. Students were then requested to make an evaluation of the conduciveness of the pre-set PBL/lecture conditions through the questionnaire (Appendix G and H). Students from the experimental group were complaining about the change in the teaching method while the students from the comparison group found their conditions suitable for the lecture method. However, with

the passage of time the experimental group found the entire teaching approach to be more enriching than they had anticipated. The experimental group further confirmed in the focus group interviews that their wish was to extend the enriching teaching approach to all the other mathematics topics and also to other science bridging course for undergraduate mathematics subjects.

(10) Extent of implementation of a student centred approach during the entire teaching-B10: PBL is said to be a student centred approach, therefore successful implementation of the PBL is also determined by the extent to which student centeredness is adhered to. The experimental group confirmed that the PBL teaching method used was student centred, practical and realistic. They further acknowledged that the teacher's attention was poorly given to them since they were the ones to do most of the work. Most students from the comparison group did not agree with the adoption of a student centred approach during the teaching and learning of compositions and inverses of functions. However, they acknowledged the usefulness of the employment of group work activities and tutorials and group projects, which also greatly influenced their understanding of compositions and inverses of functions.

(11) Students' evaluation of the sufficiency of problem solving-B11: The students from both groups also evaluated the sufficiency of problem solving during the lessons for compositions and inverses of functions. The experimental group members were satisfied with the problem solving exercise which was employed during their lessons which enhanced their understanding of functions to a greater extent (see 5.10.5). The comparison group members confirmed that they had worked through a lot of mathematical problems but what they liked most were the tutorials and group work which was a dilution of the lecture method which were included in the teaching of compositions and inverse of functions (see 5.10.5).

(12) Students' views on the effect of using real life problems in the teaching of compositions and inverses of functions-B13: One of the unique aspects of the PBL teaching approach is its ability to use real-life examples in a classroom setup. This attribute allows students to be effective problem solvers in real life and in the various workplaces. It also allows the students the meet the demands of the changing world of

work (see 5.10.6 and 5.11.6). The students from both groups acknowledged that working with real-life examples really enhanced their understanding and left them with enriching everlasting academic experiences. The experimental group members further confirmed that PBL left them with the ability not only to connect compositions and inverses of functions to daily life, but also to make connections with other related mathematical topics. This was enhanced by the fact that the formulation of a function, composition and inverses of a function from word problems in most cases required students' understanding of substitution, logarithms, exponentials or quadratics depending on the nature of the function to be formulated. Therefore prior knowledge on these topics was needed for them to understand functions taught in a PBL approach.

(13) Sufficiency of resources to support PBL/lecture method-B12: The students from both groups confirmed that they had sufficient resources to support their specific teaching method during the teaching and learning of compositions and inverses of functions. The experimental group found the provided resources PBL to be sufficient to teach compositions and inverses of functions. They also found the utilised PBL problems to be helpful in creating the necessary connections to solve problems. The comparison group also confirmed that they wouldn't have had such resources under normal circumstances and they were grateful for the supplied resources. Although there were cases of students in the comparison group who could not utilize the provided resources to make the necessary mathematical connections, they acknowledged that the resources were helpful.

(14) Evaluation of whether students' understanding of compositions and inverses of functions was as a result of their experiences with a particular teaching approach-B14: As highlighted earlier on in this chapter, the main purpose of this study was to find the effect of the PBL approach in the teaching and learning of compositions and inverses of functions. Students from the two groups had to make an evaluation as to whether they could attribute their performance in compositions and inverse of functions to chance or to the teaching method employed by the teacher. The hypothesis test performed on the post-test results on the students' performance showed that the experimental group performed significantly better than the comparison group overall (see 5.6).But in terms

of cognitive domains the only one domain showed a significant difference but the rest of the domain did not show a significant difference in performances between the two groups (see 5.8.1).

(15) Process, product and presentation of projects on compositions and inverses of functions (B15): Students from both groups did a research project based on their understanding of compositions and inverses of functions. The students were assessed on the process, product and presentation of their group project. Though the *z* test carried out on the overall project results in terms of domains the experimental group results were only significant on the process domain and the oral presentation domain but on the product domain there was no significant difference in the two groups (see 5.15).

6.4. THE EFFECT OF THE PBL APPROACH IN THE TEACHING AND LEARNING OF COMPOSITIONS AND INVERSES OF FUNCTIONS.

In this section, the researcher will address the research questions of the study. The researcher will restate the research questions which the study addressed:

1. What are the students' perceptions with regard to the relevance of inverses and compositions of functions as concepts in a subject that determines their academic destination?
2. What are the students' preferences of the presentation format for implementing the PBL approach for increased student learning outcomes in the teaching of inverses and composition of functions?
3. How do FP students experience the PBL approach in the teaching and learning of inverses and composition of functions compared to those who are taught using the lecture method?
4. How does the FP students' performance on inverses and composition of functions as a result of their PBL experience compare to those who are taught using the Lecture method FP?

With regard to students' perceptions in relation to the relevance of the concepts of compositions and inverses of functions in a topic that determines their academic ends.

The participants from both groups found the concepts to be relevant and very helpful since it is connected to many mathematical topics. Though this topic was relevant, for it needed to be supported by other topics like equations, subject of formula trigonometry, quadratic functions, logarithms and exponential functions. These should have been taught before it so that a proper prior knowledge base is set, upon which compositions and inverses of functions will be built. The students from both groups acknowledged the fact that this concept laid the relevant academic knowledge they need for their pre-calculus module in their first year at the university. These findings concur with those of Vidakovic (2004) who noted that the conceptions and reasoning patterns needed for a strong and flexible understanding of compositions and inverses of functions are more complex than what is typically assumed by the designers of curriculum and instruction. Students who think about functions only in terms of symbolic manipulations and procedural techniques are unable to comprehend a more general mapping of a set of input values to a set of output values; they also lack the conceptual structures for modelling function relationships in which the function value (output variable) changes continuously in tandem with continuous changes in the input variable (see 3.5).

Calson (2003) also asserts that the teaching of compositions and inverses of functions are also foundational for understanding major concepts in advanced mathematics (see 3.5.4). On this aspect, Dubinsky and Wilson (2004) state that for future curricular interventions to assist students in developing a robust function conception, a conception that includes a view of function as an entity that accepts input and produces output and enables reasoning about dynamic mathematical content and scientific contexts is critical. It is also important that the teaching and learning of functions should be connected to conceptual learning, so that students will be better equipped to apply their algebraic techniques appropriately in solving novel problems and tasks. It is encouraged that during the teaching and learning of compositions and inverses of functions, teachers adopt teaching methods that will lay the proper foundation needed by students to perform well in their future mathematics courses.

With regard to the second question, What are the students' preferences of the presentation format for implementing the PBL approach for increased student learning outcomes in the teaching of inverses and composition of functions, findings from the focus group discussion of this study reflected that the implementation of the PBL method should be done right at the onset of the bridging course for undergraduate mathematics, in February, until November (see 5.13.4). The experimental group students also felt that the PBL method should be implemented in all the Foundation Programme subjects. They further felt that every mathematics topic must have a research project that will allow them to come together and share ideas to solve problems. In addition to that, experimental group students also felt that the PBL approach should be used together with other conventional teaching approaches (see 5.13.4). They also felt that more time should be created for teacher-student interaction and student-student interaction during the discovery learning process.

On the other hand, those students who were taught compositions and inverses of functions using the lecture method felt that the method was good but should always be supported by tutorials, group work and projects in order for students to learn from each other rather than making it a teacher dominated learning environment. However the researcher acknowledges the dilution of the lecture method through the use of the research projects and group works which were strength on the teaching and learning of the students but cited as limitation for the study.

With regard to the third question, on how students taught using the two methods compare in terms of their experiences, this study found out that those students who were taught using the PBL approach performed significantly better than those who were taught using the lecture method. This was probably because the PBL approach is more of a student centred learning approach, where students are the ones to discover their own learning while the learning that takes place in a lecture environment is more teacher centred. The way the students experienced their learning was different because of the teaching method approaches. However, the researcher tried to make the learning environments comparable by introducing group work, tutorials and group projects to both groups. Some students from the comparison groups felt that the lecture method's role was just to push the syllabus with little focus on whether students benefited or not. The PBL group felt that PBL was meant to make them to be in total comparison of the learning that takes place in the classroom. They felt that they had the overall ownership of the entire learning process (perceived learning as something they have to do) than the lecture group which perceived learning as something that must be done to them. This variation in student' experiences might have contributed to the variations in the performances of the students in the two groups.

With regard to the fourth question, on how the performances of the two groups of students compared, there were similarities and differences in terms of students' performances. A two tailed z hypothesis for the students' performances on the overall results at post-test showed that the PBL group performed significantly better than the lecture group, but in terms of performance by domain the significant difference was noted in only one of the five domains which were used in this study. The two-tailed z hypothesis test on overall results of the presented projects also confirmed the superiority of the PBL group's performance though in terms of domains only two were significant. Based on these findings, the performances of the experimental group was significantly different from the comparison group but the role of the lecture approach cannot be undermined since the results from the two groups were fairly comparable. Therefore based on these findings, it can be concluded that the PBL can help to improve performances in some domains of compositions and inverses of functions but it does not show significant difference in some domains. These results further suggest that PBL has its role in the teaching and learning of compositions of functions but the lecture method also has a role. Therefore the combined use of the two teaching approaches in the teaching and learning of functions would benefit students more.

To summarize this section, the participants in this study did not significantly different performances, with the PBL group performing better than the comparison group in the overall pre-post test results analysis (see 5.9). But in terms of HLT domains PBL did not prove to be more efficient in the learning of compositions and inverses of functions. Though the PBL group seemed to have performed better than the lecture group in the overall pre-pre-post-test analysis, the average performances of the lecture group students was also above 60 percent for both the post-test and the presented projects which showed that this method was also effective in improving students performances (see 5.9). Therefore, one can claim that the PBL approach has the potential to improve students' performance in some HLT domains but the role of the lecture method cannot be overruled completely in the teaching and learning of composition and inverses of functions. Conclusively, it can be claimed that both PBL and the lecture approach play significant roles in making students understand the HLT domains of compositions and inverses of functions and should always be used together.

In the next section the researcher presents the research gap which this study intended to fill in the field of mathematics and mathematics education.

6.5. THE RESEARCH GAP

This study will contribute to improved PBL design and implementation in the teaching and learning of composition and inverses of functions, more especially in an African FP setting. Most importantly, it might probably make significant improvement to the existing theory with respect to emerging problems associated with the teaching of composition and inverses of functions using the PBL approach in an African educational setting. The students' experiences with the PBL approach were documented in the UNAM setting, and a domain specific PBL Hypothetical Learning Trajectory (HLT) was designed (see chapter 4).

6.4.1 Contributions in terms literature review four different concepts

In chapter 3, the researcher referred to the limited available literature on the concept of inverses and compositions of functions. Nguyen (2009) looked at the implementation of PBL on undergraduates in Vietnam, and Gibbings (2008) did a study on students and PBL in a virtual space which was a phenomenographical study that involved distance learning online engineering students. Tiwari (1998) investigated the effect of PBL on students' critical thinking dispositions and approaches to learning: a study which was done on nursing students. Vidakovic (2006) did a study on students' understanding of calculus, whereas Lucas (2002) investigated on compositions and inverses of functions: main ideas as perceived by teachers and pre-service teachers. Moreover, Dubinsky & Harel (2004) focused on compositions of functions with undergraduates. Even (1993) on her study on teachers' content knowledge on functions stated that many studies which were done on students' understanding of compositions and inverses of functions reported little understanding of these concepts by both teachers and students. Apart from that, there are many studies that investigated on students' understanding of function situations for the student to (Even, 1993; Kimani, 2008; Cotril, 1999; Dubinsky & McDonald, 2001; Lucus, 2003). Typically these studies ask students to give their difinitions of functions,and then present many situations involving functions to the students. None of these studies employed the PBL approach in the teaching of compositions and inverses of functions focusing specifically in a bridging course for undergraduate mathematics. In addition, Dubinsky and Wilson (2009) focused on compositions of functions only; whereas Kimani (2008) did studies on undergraduate students' understanding of compositions and inverses of functions. Kimani (2008) further argue that

> "while there have been many research studies on the teaching of functions, teachers understanding of functions ideas, there is still a lack of research on teaching and understanding with regard to function transformation, function composition and function inverse………"(p.45-45).

The quotation above support the claim that this study fills an important knowledge gap in the teaching and learning of compositions and inverses of functions. Based on the above, it can be seen that there are several studies that have been done on PBL, but studies focusing specifically on (compositions and inverses) are very limited if any, especially targeting FP students. This study makes a modest contribution to literature marrying five different concepts (foundations programme, PBL, compositions functions, inverses of functions and functions as a mathematics topic) which makes it a unique study in a bridging course for undergraduate mathematics.

To summarize this section, the theoretical contributions of this study is through adding new knowledge on teachers' implementation of PBL in the teaching of compositions and inverse of functions in a FP and the introduction of PBL in a Namibian classroom environment. Another contribution is on the teaching of functions in general and to other population groups which might want to learn the same mathematical content. This study also made significant contributions in three major dimensions of confirmatory, explanatory and action research dimensions.

6.4.2. Methodological contributions

This study used questionnaires, focus group interviews, observation schedules and projects as methods of data collection. Most of the questions were designed by the researcher and they were not coming from the students' study manuals. Therefore, these resources which were gathered can always be utilized by other lecturers to enrich their students with intensive knowledge on compositions and inverses of functions taught following the PBL approach.

Most of the PBL approaches that have been done so far either makes use of the PBL shoestring approach or Burrows' seven jump approach. The shoestring model is used over three or more years and PBL is applied to different year level students who pass through different lecturers. In this study a modified shoestring model was used to suit the SFP mathematics and this was coupled with Barrows seven jump approach to form a new hybrid model which is suitable for a SFP like the one at Oshakati UNAM. The implementation of the hybrid PBL model, helped to strengthen students' understanding of composition and inverses of functions taught using the PBL approach in a SFP. Therefore, this combination of modified PBL models can also be used by other researchers who may intend to implement PBL in their teaching. In this study the element of exposing both groups to group work, projects, etc. to both groups in order not to unduly disadvantage the comparison or lecture group lead us to a hybrid model of applying PBL.

Another contribution which is methodological in this study was the employment of a diluted lecture method to teaching and learning of compositions and inverses of functions through the use of tutorials, group works and research projects applied to foundation students. Under normal circumstances these are not usually used. Therefore this diluted lecture approach which made significant differences in students' performances in the teaching and learning of compositions and inverses of functions can also be used by other researchers who are in the similar educational settings in future.

6.6. RECOMMENDATIONS OF THE STUDY

The findings of the study reflect that PBL is an effective teaching approach which has the potential to improve foundation students' understanding of compositions and inverses of functions. In this study all the students who were taught using the PBL approach considered it as a worthwhile approach for the teaching and learning of not just composition and inverses of functions but functions and other mathematics topics in general. Therefore, this study suggests that PBL could be introduced on a wider range of mathematics topics and other FP subjects in order to diversify teaching approaches and thereby improve students' understanding. All the students who were taught using the PBL approach commented positively on the effectiveness of the approach and showed their desire to have opportunities to learn more through the approach.

I hope to disseminate my study findings to other interested bridging course for undergraduate mathematics administrators in university FP and schools to demonstrate how a PBL was implemented at the Oshakati Science SFP university setting. As a result of this, I hope that FP managers and school administrators would encourage a large number of University lecturers and school teachers to implement PBL in their teaching, as well as providing a wide range of support for teachers and lecturers in the implementation of PBL in the Namibian education system.

Apart from that, the researcher strongly recommends the implementation of PBL, not only in the teaching of compositions and inverses of functions, but also in other disciplines of the University. Historically, PBL was introduced into medical education and other health sciences prior to its introduction to applied sciences and social sciences. This study confirmed positive responses from students in mathematics to the PBL approach, a new area for consideration in the 21st century.

The Namibian educational policy makers should continue to encourage teachers and support them to build their skills in PBL. If the Namibian Ministry of education is to implement PBL across various schools, there must be an administrative teacher support panel for PBL implementation. Burrows and Tumbling (2003), state that the availability of an administrative support is an important factor to consider for a successful PBL implementation in a school/university setting. On this regard, the researcher intends to publish the findings of this study in international journals and publications in order to argue for the effective impact of PBL in Namibian settings to a larger number of Namibian educators.

The implementation of the PBL involves a wide range of methods that will increase the interaction and communication between students and teachers/lecturers, as well as student to student, and therefore it is important to enhance students' collaboration and build a sound team-working spirit among them. The researcher advises that further administrative support in the form of ICT must be given to enable students and teachers to effectively communicate and be engaged in the teaching and learning using a PBL approach in Namibia.

In addition to that, the administrative support should be provided to the teachers/lecturers to implement authentic practices that support the PBL approach to teaching and learning. The focus should be to move away from focus tests and implement assessment system changes that will allow the measurement of learning objectives using a variety of processes. In the long run, the researcher wishes to avail the findings of this study through presentations in conferences at SADC level or internationally; on the implementation of the PBL approach in mathematics so as to improve the teaching and learning of mathematics in the region.

The Namibian ministry of education is gradually making changes in mathematics syllabi and also changing the teaching approaches so as to meet the professional demands and expectations of the current job market. The researcher is expecting to contribute to these current changes through the findings of this study. The researcher strongly suggests making changes more effective especially when the Namibian educators systematically recognize all factors in the teaching and learning process such as the importance of having all steps in the process including consistency in teaching and assessment styles.

Findings of this study have broadened the researcher's understanding of the PBL and he wishes to share these enriching experiences with other bridging course for undergraduate mathematics lecturers and school teachers through seminars. The researcher believes that the gained experiences will help to improve and refine his own teaching and the teaching for those he will share his PBL experiences with.

One of Namibia's vision 2030 objectives as already noted in the introductory chapter (see 1.1) (iv) is to transform Namibia into an industrialized nation of equal opportunities, which is globally competitive, realizing its maximum growth potential on sustainable basis, with improved quality of life. The realization of this objective is not possible without an improved educational system that teaches students using effective teaching approaches like PBL. However, in order to effectively implement and innovate using a challenging teaching approach like PBL, Namibian policy makers, teachers and lecturers have to first change their perspectives and beliefs about the use of traditional teaching approaches. It is quite challenging for school teachers and university lecturers to change their perspectives of the teacher being seen as an authority figure in the classroom and move towards empowering students as learners who can construct their own knowledge.

6.7. POSSIBLE AREAS FOR FURTHER RESEARCH

There are several areas where this study presents certain limitations which the researcher suggests that there can be opportunities and areas for further research.

The focus of the study was on the effect of a PBL approach in the teaching and learning of compositions and inverses of functions in a SFP. This study only focused on science foundation students based at Oshakati campus. It would be good to explore the effect of the PBL approach in different campuses and compare the students' experiences across campuses. Apart from that, this study only implemented PBL in one mathematical concept (compositions and inverses) of functions. Research needs to be done to find out how students will experience PBL in other mathematical topics so that the students' experiences can be compared across foundation mathematics topics.

This study applied PBL only in a mathematics context; research needs to be done to find out how students will experience PBL across other Foundation Programme subjects such as Biology, Physics, Chemistry and English and then correlate students' PBL experiences across subjects and across campuses.

The topic composition and inverses of functions is taught in the FP, which is a programme that prepares students for university mathematics. It is assumed that the composition and inverses of functions that are taught at FP are sufficient to prepare students for University mathematics but studies need to be done to find out the extent to which the compositions and inverses of functions taught at FP prepares students for University basic Mathematics, Pre-calculus, Calculus and other related mathematics modules.

In this study PBL effects were more pronounced in effects and learning strategies than in achievement. What could be interesting for further investigation would be the question of relative proportions of PBL and lecture approaches in deployment in order to produce optimal complementary roles.

It is arguable that in the case of FP, compositions and inverses of functions it is enough for students to possess a strong procedural knowledge at least for high school grade 12. My opinion is completely different, therefore further research needs to be undertaken to find out how the teaching and learning of composition and inverses of functions beyond procedural fluency could impact on students' performances in bridging course for undergraduate mathematics.

6.8. LIMITATIONS OF THE STUDY

The successful completion of this study might have been affected by time limitations since the researcher is a fulltime lecturer who has a full workload to teach. Performing PBL experiments needed a lot of time and commitment from both the researcher and the FP students which might have been compromised on the study time for the Science Bridging course for undergraduate mathematics students. This study employed focus group interviews, questionnaires and observation protocols as part of its data collection procedures. Therefore the researcher is aware of the fact that findings from qualitative data may not be generalised to other contexts. In this study the employment of group work in the comparison group was a dilution of the lecture method and yet a strength for the comparison group. The limited duration (one year) of the exposure of students to PBL could have constrained a comprehensive understanding and adherence to the approach's fundamental principles.

6.9. CONCLUSION

The implementation of PBL in FP learning has the potential to improve students' understanding of mathematics. This section gave a recap of the chapters of the study, a summary of the results of the study followed by an address of each of the research questions for the study. This chapter then gave the study gap which this research intended to fulfil, followed by the recommendations of the study and possible areas for further research based on the perspectives that emanated from the findings of the study.

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APPENDIX A: PRE-TEST POST-TEST FOR BOTH GROUPS

Department of Mathematics University of Namibia, SFP August 2013

Topic: Functions **Test 1:** Pre and Post Test

Duration: 90 Mins **Researcher: M Chirimbana**

Total 70

Instructions candidates

Answer all questions. Try to understand each question before you make an attempt to answer it. Number the questions clearly and present your solutions in a logical manner. Show all the necessary working.

1.a) Give the definition of a function giving real world examples [2]

b) Your computer's screen saver is an expanding circle. The circle starts as a dot in the middle of the screen and expands outward, changing colours as it grows. With a 51 cm screen, you have a viewing area with a 5cm radius (measured from the centre diagonally down to a corner). The circle reaches the corners in four seconds. Express the area of the circle (discounting the area cut off by the edges of the viewing area) as a function of time t in seconds. [10]

2. a) Explain what you understand by the term function? [2]

b.) When do we say a function is one-to-one? [2]

c.) Give an example of a one-to-one function [2] d.) Use a diagrammatic representation to show a function [2]

3.a) When do we say a function is a composition function?

[3]

b) Give an example of a composition function? [3]

c.) John is your brother and cannot see, neither can he read nor write but he can hear. One day when he was going to church he heard some boys from the neighbourhood boosting to each other about composition functions after their grade twelve mathematics teacher had taught them. John wants to know what composition functions are, if you were one of these boys how would you explain to John what composition functions are.

[10]

4.) The function f is given by $f : x \rightarrow \dfrac{3}{x+2}; x \neq -2$

i. Evaluate $ff^{-1}(4)$
ii. Obtain and expression in terms of *x* for the inverse of *f.*
iii. What type of a function is f?
iv. Given that $f(a) = 2a + 3$ find the value of a.

[1, 2,1,2]

5. What do you understand by evaluating the inverse of a function; give a clear illustration on when one can evaluate the inverse of a function**.**

[2]

6. During a sale a music drum kit reduces their price by 20%. Preferred customers also received a further 15% discount.

a) Write a composition function to represent the final cost price for a preferred customer on a kit that was originally costing $c.

[2]

b) Find the cost of a drum kit priced at $248 and a preferred customer wants to buy it. **[2]**

7. C(t) gives the cost of cooling a house in a desert when the average daily temperature is T^0C. T(d) gives the number of floors of stairs an average person can walk in t minutes.

a) What composition function would you need to use to determine the number of calories to be burnt in a certain time?

[2]

b) How many calories would you need to burn in 30 minutes? **[2]**

8. Fig 2 below shows the graph of the function $y = f(x)$.Sketch the graph of the inverse of *y* = *f(x)* on the graph and justify your sketch.

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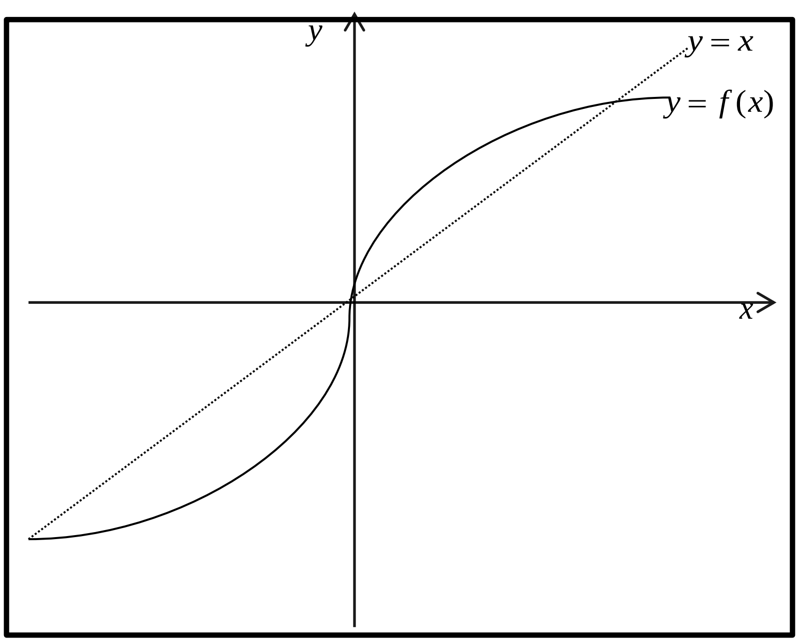

[2]

9. A shop owner needs to prize a digital camera for a customer. The customer paid a total of $103.14 which includes a $3 gift wrapping and an 8% fax charge. What prize must the shop owner put on the price at? **[2]**

10**.** The function f and g are defined by $f : x \to 3x + 4;\ \ g : x \to \frac{1}{2}x;$

i. Evaluate $ff^{-1}(-3)$

ii. Obtain and expression in terms of x for the inverse of *fg(x)..*

iii. What types of a functions are *f* and *g*?

iv. Write down $f^{-1}\ and\ g^{-1}$ in a similar form and show that $g^{-1}f^{-1} = (fg)^{-1}$

v. Given that $f(a) = 2a + 3$ find the value of a.

[2, 2, 2, (2,2, 2), 2]

APPENDIX B: LESSON/TUTORIAL OBSERVATION PROTOCOL PBL/LECTURE GROUP

Department of Mathematics University of Namibia, SFP August 2013

Topic: Functions Observation points

Duration: 60 Mins ……… Lecturer: M Chirimbana

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1. How does the lecturer commence the lesson?

2. How does the lecturer use problems used as a starting point during the tutorial session?

3. What is the nature of the problem (s) given to learners in the tutorial session?

4. How learners are divided into meaningful study groups?

5. What roles are assigned to learners in their different groups during tutorial sessions?

6. How do the learners play out their role to achieve their task objectives in their different groups?

7. How do the learners give feedback to the lecturer/ tutor on their group problem solution strategies in the tutorial?
8. What is the role of the lecturer in the tutorial?

APPENDIX C: Interview Guide: Students' perceptions and experiences about the teaching and learning of composition and inverses of functions using PBL

This section seeks to elicit your perception and experiences about the teaching and learning of composition functions and inverses of functions at FP

Date of Interview: …………………… Venue of Interview:……………………

Interview starting time:………… Interview ending time :...(50-60)minutes

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This Interview guide has four sections: Section A, Section B, and Section C and Section D

Section A: General teaching and learning approaching

Section B: Students' perceptions about the teaching and learning of composition and inverse functions at FP

Section C: Students' PBL/Lecture experiences in the teaching and learning of composition and inverses of functions at FP.

Section D: Students Recommendations by students

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1. **Reminders to Interviewer: Before the interview, the interviewer must remember the following:**

i. Identify the person/persons to be interviewed.

ii. Arrange and agree on the interview venue.

iii. Remember to read those perception responses that require probing.

iv. Explain to the interviewees about the purpose of the interview.

v. Assure the interviewees of confidentiality and anonymity.

vi. During the interview, the interviewer must first Introduce himself

vii. Request for permission to record the interview proceedings.

viii. Remember to thank the interviewees for their enriching information

ix. Remember to clarify to the interviews that they are free to ask for clarifications on questions they don't understand.

x. Remember to probe and stimulate the interviewees, explaining to them on related teaching methods.

Section A: General teaching and learning of composition and inverses of functions

A1:

*****In the questionnaire guide you just answered, questions related to composition and inverses of functions, what are your experiences with regard to the teaching of composition and inverses of functions function (Probe for more).

*****What aspects did you enjoy learning when you were doing composition and inverses of functions?

*Were the offered tutorials relevant to your studies ?(probe for more basing on the given answer)

*How helpful were the group activities you were doing? (probe for more basing on the given answer

---

Section B: Questions related to students perceptions about the teaching of composition and inverses of functions using PBL/Lecture method

A2:

*****What are your perceptions with regard to the teaching and learning of composition and inverses of functions at FP?

***** What are your perceptions about the teaching of compositions and inverses of functions at FP using the method your teacher used?

*****What is your perception with regard to the type of questions your teacher used to ask? (probe for more for relevancy and applicability to composition and inverses of functions and the type of teaching employed by the teacher.

Section C: Questions related to students personal experiences with composition functions and inverses teaching methods (PBL/Lecture).

A3:

***** What do you count as being the important aspect in the teaching and learning of composition and inverses of functions? (probe for more)

*During your time of learning of composition and inverses of functions how did you know that you have fully understood/mastered a particular concept? (Probe for more).

*How do you feel about the teaching and learning of composition and inverses of functions using the lecture method/ using the PBL? (probe for more)

*From your own experiences, what are the qualities of a good tutorial session? (probe)

*What do you think are the most challenging aspects when it comes to the teaching and learning of composition and inverse functions? (probe for more)

*If you were to be put in a tutorial group, what would be your role within the group? (probe for more)

*What is the role of problem based learning in the teaching and learning of composition functions and inverses, and how can this learning approach best be implemented in the mathematics foundation class to improve students' performances? (probe)

Section D: Students Recommendations by students.

D1:

***** If you were to be provided with an opportunity to make improvements at the SFP, what changes would you make in the teaching and learning of compositions and inverses of functions to improve the passing of students (probe)?

*****Can you share with me your best study strategies that improved your understanding of composition and inverses of functions at the FP?(probe).

*What advice would you give to new friends who have joined the Mathematics FP class for the first time for them to improve their performance (probe)?

APPENDIX D: Interview Guide: Students' perceptions and experiences about the teaching and learning of composition and inverses of functions using Lecture approach.

This section seeks to elicit your perception and experiences about the teaching and learning of composition functions and inverses of functions at FP

Date of Interview: …………………… Venue of Interview:……………………

Interview starting time:………… Interview ending time :...(50-60) minutes

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This Interview guide has four sections: Section A, Section B, and Section C and Section D

Section A: General teaching and learning approaching

Section B: Students' perceptions about the teaching and learning of composition and inverse functions at FP

Section C: Students' Lecture experiences in the teaching and learning of composition and inverses of functions at FP.

Section D: Students Recommendations by students

---

2. **Reminders to Interviewer: Before the interview, the interviewer must remember the following:**

xi. Identify the person/persons to be interviewed.

xii. Arrange and agree on the interview venue.

xiii. Remember to read those perception responses that require probing.

xiv. Explain to the interviewees about the purpose of the interview.

xv. Assure the interviewees of confidentiality and anonymity.

xvi. During the interview, the interviewer must first Introduce himself

xvii. Request for permission to record the interview proceedings.

xviii. Remember to thank the interviewees for their enriching information

xix. Remember to clarify to the interviews that they are free to ask for clarifications on questions they don't understand.

xx. Remember to probe and stimulate the interviewees, explaining to them on related teaching methods.

Section A: General teaching and learning of inverses and compositions of functions.

A1:

*****In the questionnaire guide you just answered, questions related to composition and inverses of functions, what are your experiences with regard to the teaching of inverses and compositions of functions (Probe for more).

*****What aspects did you enjoy learning when you were doing compositions and inverses of functions?

*Were the offered tutorials relevant to your studies (probe for more basing the given answer)

*How helpful were the group activities you were doing (probe for more basing the given answer

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Section B: Questions related to students perceptions about the teaching of composition and inverses of functions using Lecture method

A2:

*****What are your perceptions with regard to the teaching and learning of compositions and inverses of functions at FP?

***** What are your perceptions about the teaching of compositions and inverses of functions at FP using the method your teacher used?

*****What is your perception with regard to the type of questions your teacher used to ask? (probe for more for relevancy and applicability to inverses and compositions of functions and the type of teaching employed by the teacher.

Section C: Questions related to students personal experiences with compositions and inverses of functions taught using the Lecture method

A3:

***** What do you count as being the important aspect in the teaching and learning of composition and inverses of functions (probe for more)

*During your time of learning of inverses and compositions of functions how did you know that you have fully understood/mastered a particular concept? (Probe for more).

*How do you feel about the teaching and learning of inverses and compositions of functions using the lecture method (probe for more)?

*From your own experiences, what are the qualities of a good tutorial session (probe)?

*What do you think are the most challenging aspects when it comes to the teaching and learning of compositions and inverses of functions (probe for more)?

*If you were to be put in a tutorial group, what would be your role within the group (probe for more)?

*What is the role of lecture method in the teaching and learning of compositions and inverses of functions, and how can this learning approach best be implemented in the pre-calculus class to improve students' performances (probe)?

Section D: Students Recommendations by students.

D1:

***** If you were to be provided with an opportunity to make improvements at the bridging course for undergraduate mathematics what changes would you make in the teaching and learning of compositions and inverses of functions to improve the passing of students? (probe)

*****Can you share with me your best study strategies that improved your understanding of inverses and compositions of functions? (probe).

*What advice would you give to a new friend who has joined the Mathematics FP class for the first time for them to improve their performances? (probe)

APPENDIX E: Assessment Rubric for group presentations for compositions and inverses of functions research problem.

The researcher will use the following rubric to assess the presented group projects work.

1.Your process	**Evidence**	**Marks**
Your sources of inspiration of the problem (showing an ability to **create**)	Project theme and interview	**/5**
Problem is real and relevant to the theme "composition and inverses of functions **"(Showing the art of creativity)**	Group Problem statement	**/5**
Your group generates a variety of ideas and gives convincing reasons for deciding on the problem. Showing the ability to **analyse**	Individual *my problem log*	**/10**
Problem solving tasks are well defined and good and logical thinking follow up questions are asked. Showing the ability to **understand**.	*Each one teach one* worksheets	**/5**

2.The Product	**Evidence**	**Marks**
Your group shows that you have made thorough use of knowledge from the content of the theme" compositions and inverses functions". Showing **understanding**	Oral presentation	**/5**
Your group has carried out some experiments/data collection procedures in trying to solve the problem. Showing an ability to **analyse**	Oral presentation	**/15**
Your group findings were well- **analysed,** interpreted and reported (Showing an ability to **evaluate**)	Oral presentation	**/10**
Your explanatory brief is clear and comprehensive and explains well all the steps that you have taken to solve the problem. Showing **understanding**.	Oral presentation	**/5**
There is a high level of **creativity** in your ideas and approaches, logical solutions, and wise usage of academic resources.	Oral presentation	**/10**

3. Oral presentation	**Evidence**	**Mark**
Organisation of group the presentation	Oral presentation	**/5**
Ideas and findings are well communicated **(Well understood and remembered)**	Oral presentation	**/5**
IT/Visual aids used are effectively used(**Creativity ability)**	Oral presentation	**/10**
Group ability to answer the questions from the audience. Showing clear **understanding**	Oral presentation	**/10**

Group Name	Members	Title of problem	Process	Presentation	Total Scores

Project assessment criteria Adopted from: (Guba, 1981)

/100

Total Score

APPENDIX F: Group Project (PBL /Lecture)Groups

Department of Mathematics University of Namibia, SFP August 2013

Topic: Inverses and compositions of functions Total 100 Marks

Duration: 3 Months Examiner: M Chirimbana

Instructions candidates

Answer all questions. Try to understand each question before you make an attempt to answer it. Number the questions clearly and present your solutions in a logical manner. Show all the necessary working.

Question 1 below is a group research activity.

1 Design a project on inverses and compositions of functions and explain how your project is connected to composition and inverses of functions you may consult the library for more information and make sure all the consulted sources are included in the references. Prepare a portfolio for your findings.

You may consider using some or all of the following guidelines:

Process

- Your sources of inspiration of the problem (What inspired/motivated the group to research on the problem)
- Problem is real and relevant to the theme composition and inverses of one-to-one functions.

- Your group generates a variety of ideas and gives convincing reasons for deciding on the problem solving tasks are well defined and good and logical thinking follow up questions are asked.
- Evidence of collected data with well stipulated dates.

The Product

- Evidence that the group shows that they have made thorough use of knowledge from the content of the theme composition and inverses functions.
- Evidence that the group has carried out some experiments/data collection procedures in trying to solve the problem.
- Clear evidence that the group findings were well- analysed, interpreted and reported
- Brief and clear explanations and comprehensive and well explained steps that you have taken to solve the problem.
- Level of creativity in group ideas and approaches,
- Logical solutions and wise usage of academic resources.

Oral Presentation

- Organisation of group presentation.
- Group ideas and findings are well communicated to the audience.
- IT/Visual aids used are effectively used during the presentation.
- Group ability to answer the questions from the audience.

[100]

APPENDIX G: PBL GROUP Questionnaire on Students' perceptions and experiences of PBL about the teaching and learning of the compositions and inverses of functions

This section seeks to elicit your perception and experiences about the teaching and learning of compositions and inverses of functions.

Section A: Demographic information

Section B: General questions about students' experiences about the teaching and learning of compositions and inverses of functions.

Section C: General teaching and learning of compositions and inverses of functions.

SECTION A: DEMOGRAPHIC INFORMATION

A1. What is your gender? (Tick the correct box)

Male	Female
1	2

A2.What is your age to the nearest year

<17	17-18	18-19	20-21	>21
1	2	3	4	5

SECTION B

PART 1: Topic: Functions

B1: What aspects of compositions and inverses of functions did you enjoy most during the course of your study?

…………………………………………………………………………………………………
…………………………………………………………………………………………………
………………………………………………………………………………………………..

B2. What aspects of compositions and inverses of functions did you find most challenging during the course of your study and why?

…………………………………………………………………………………………………
…………………………………………………………………………………………………
…………………………………………………………………………………………………

B3: The tutorial sessions were helpful to make me understand compositions and inverses of functions?

Strongly Agree	**Agree**	**Disagree**	**Strongly disagree**
1	2	3	4

B4: Working in groups meant that you learnt from one another in doing compositions and inverses of functions?

Strongly Agree	**Agree**	**Disagree**	**Strongly disagree**
1	2	3	4

B5: Sufficient conditions were created to necessitate PBL during the lessons during the teaching and learning of compositions and inverses of functions.

Strongly Agree	**Agree**	**Disagree**	**Strongly disagree**
1	2	3	4

B6: Sufficient guidance was given by the teacher during the teaching of compositions and inverses of functions.

Strongly Agree	**Agree**	**Disagree**	**Strongly disagree**
1	2	3	4

B7.I learned as much problem solving I would have done in a conventional lecture when you were doing compositions and inverses of functions?

Strongly Agree	**Agree**	**Disagree**	**Strongly disagree**
1	2	3	4

B8.: Considering the material I had, when I was doing compositions and inverses of functions I think I would have had as much in a conventional lecture

Strongly Agree	Agree	Disagree	Strongly disagree
1	2	3	4

B9: Focusing the teaching of inverses and compositions of functions to real life problem made the topic more interesting and relevant.

Strongly Agree	Agree	Disagree	Strongly disagree
1	2	3	4

B10: I now understand the compositions and inverses of functions?

Strongly Agree	Agree	Disagree	Strongly disagree
1	2	3	4

APPENDIX H: COMPARISON GROUP Questionnaire on Students' perceptions and experiences of Lecture method in the teaching and learning of the inverses and compositions of functions

This section seeks to elicit your perception and experiences about the teaching and learning of compositions and inverses of functions.

Section A: Demographic information.

Section B: General questions about students' experiences about the teaching and learning of compositions and inverses of functions.

Section C: General teaching and learning of compositions and inverses of functions.

SECTION A: DEMOGRAPHIC INFORMATION

A1. What is your gender? (Tick the correct box)

Male	Female
1	2

A2.What is your age to the nearest year

<17	17-18	18-19	20-21	>21
1	2	3	4	5

SECTION B

PART 1: Topic: Functions

B1: What aspects of composition and inverses of functions did you enjoy most during the course of your study?

……
……
………………………………………………………………………………………………………..

B2. What aspects of compositions and inverses of functions did you find most challenging during the course of your study and why?

…………………………………………………………………………………………………
…………………………………………………………………………………………………
…………………………………………………………………………………………………

B3: The tutorial sessions were helpful to make me understand compositions and inverses of functions?

Strongly Agree	**Agree**	**Disagree**	**Strongly disagree**
1	2	3	4

B4: Working in groups meant that you learnt from one another in doing inverses and compositions of functions?

Strongly Agree	**Agree**	**Disagree**	**Strongly disagree**
1	2	3	4

B5: Sufficient conditions were created to necessitate lecture method during the lessons during the teaching and learning of inverses and compositions of functions.

Strongly Agree	**Agree**	**Disagree**	**Strongly disagree**
1	2	3	4

B6: Sufficient guidance was given during the teaching of inverses and compositions of functions.

Strongly Agree	Agree	Disagree	Strongly disagree
1	2	3	4

B7.I learned as much problem solving I would have done in a conventional lecture when you were doing inverses and compositions of functions?

Strongly Agree	Agree	Disagree	Strongly disagree
1	2	3	4

B8.: Considering the material I had, when I was doing compositions and inverses of functions I think I would have had as much in my normal conventional lectures.

Strongly Agree	Agree	Disagree	Strongly disagree
1	2	3	4

B9: Focusing the teaching of compositions and inverses of functions to real life problem made the topic more interesting and relevant

Strongly Agree	Agree	Disagree	Strongly disagree
1	2	3	4

B10: I now understand the compositions and inverses of functions?

Strongly Agree	Agree	Disagree	Strongly disagree
1	2	3	4

APPENDIX I: TUTORIAL 1 PBL/LECTURE GROUP

Department of Mathematics University of Namibia, SFP
August 2013 Topic: Functions
Total 20

Duration : 60 Minutes Examiner: M Chirimbana

Instructions candidates

Answer all questions. Try to understand each question before you make an attempt to answer it. Number the questions clearly and present your solutions in a logical manner. Show all the necessary working.

GP A. The cost of printing wedding invitation cards is partly constant and partly varies as the number of cards printed, If the cost of 50cards is \$24 and the cost of printing 100 cards is \$46.50.

i. Formulate a function of C in terms of n.
ii. Find the cost of printing 80 cards.
iii. Write down and interpret the inverse of the function C(n).
iv. Represent the function C(n) and its inverse on the same graph using the definition of an inverse of a function.

GP B. The formula $K(C) = C + 273$ converts Celsius temperature to Kelvin.

The formula $C(F) = \frac{5}{9}(F - 32)$ converts Fahrenheit temperature to Celsius.

a. Write a composite function that will convert Fahrenheit temperature to Kelvin.

b. Convert the boiling point of water (212º F) and the freezing point of water (32º F) to Kelvin.

GP C. given the expression $f(x) = \frac{6x+5}{x-4}$ to find

i) The inverse of f(x).

ii.) Draw the graph of f(x) and its inverse on the same Cartesian plane

iii) How are the two graphs related to each other geometrically?

GP D. You make a purchase at a local hardware store, but what you've bought is too big to take home in your car. For a small fee, you arrange to have the hardware store deliver your purchase for you. You pay for your purchase, plus the sales taxes, plus the fee. The taxes are 7.5% and the fee is $20.

h. Write a function t(x) for the total, after taxes, on the purchase amount x.
i. Write another function f(x) for the total, including the delivery fee, on the purchase amount x.

j. Calculate and interpret (f o t)(x) and (t o f)(x).
k. Which results in a lower cost to you? Suppose taxes, by law, are not to be charged on delivery fees.

l. Which composition function must then be used?
m. Find the inverse of (f o t)(x)

GP E: In the mail, you receive a coupon for $5 off of a pair of jeans. When you arrive at the store, you find that all jeans are 25% off. You find a pair of jeans for $36.

i. If you use the $5 off coupon first, and then you use the 25% off on the remaining amount, how much will the jeans cost?
ii. If you use the 25% off first, and then you use the $5 off on the remaining amount, how much will the jeans cost?
iii. How will the situation change for jeans that cost more or less than $36?
iv. Let the cost of the jeans be represented by a variable x. Write a function f(x) that represents the cost of the jeans after the $5 off coupon.
v. Write a function g(x) that represents the cost of the jeans after the 25% discount.

vi. Write a new function r(x) that represents the cost of the jeans if the 25% discount is applied first and the $5 off coupon is applied second.

vii. Write a new function s(x) that represents the cost of the jeans if the $5 off coupon is applied first and the 25% discount is applied second.

GP F: You are an 18 year old high school student. You need to decide whether you will continue with your education or beginning working. No matter what you decide, you must pay for your expenses (variable) (food, cell phone, car, bills, power etc.).The benefit of working after school is that you begging earning money immediately. If you continue with your education, you will go to the college and pay for your own education along with all the other variables you may need to make things work for you.

THE END

APPENDIX J: PERMISSION REQUEST LETTER TO UNAM

University of Namibia Oshakati Campus

P.O. Box 2654

Oshakati

moseschirimbana@gmail.com

15 April 2013

The Director

University of Namibia Oshakati Campus

Oshakati Campus

Oshakati

Ref: Request for permission to do a research study

I am Moses Chirimbana, a member of the University of Namibia lecturing staff based at Oshakati Campus lecturing in the Science Foundation programme. I'm registered as a PHD candidate at The University of Stellenbosch undertaking a PHD in mathematics education. My student number is **15933822.** My area of focus is on Problem Based Learning Approach (PBL). The topic for my study is:

"The effect of the Problem Based Learning (PBL) approach in the teaching of compositions and inverses of functions in a Foundation Programme.

I wish to find out how the PBL approach can affect students' mathematics performance at the SFP. The overall goal is to come up with a hypothetical framework for teaching inverses and composition of one to one function using the PBL approach in the FP. Therefore I am seeking your permission to undertake this study at Oshakati Campus

Thank you in advance

Chirimbana Moses

APPENDIX K: UNIVERSITY OF NAMIBIA

NORTHERN CAMPUS

P.O. Box 2654, Oshakati, Namibia, Eliander Mwatale Street

Telephone: (++264) (65) 2232287, Fax: (++264) (65) 2232271

22 April 2013

To: Mr Moses Chirimbana

Student number: **15933822**

REF: REQUEST TO DO RESEARCH AT UNAM OSHAKATI CAMPUS (SCIENCE FOUNDATION PROGRAMME), FOR THE PERIOD OF: 1 JULY TO 1 DECEMBER 2013

Dear Sir

We hereby grant you permission to conduct research at Unam Oshakati Campus (Science Bridging course for undergraduate mathematics on the topic "**The effect of the PBL approach in the teaching of composition and inverses of functions in a Bridging course for undergraduate mathematics.** We however would like to learn from your findings, so a copy of the report will be highly appreciated. May we also request that the research activities may not interrupt with the University teaching programme

Please take a copy of this letter when you go and conduct your research as proof of permission granted.

Yours Truly

……………………….

LEENA LAHJA TILENI Nghipandulwa

COORDINATOR:SCIENCE FOUNDATION PROGRAM (OSHAKATI CAMPUS) **University of Namibia**

Tel: 065-2232287 - E-mail: lnghipandulwa@unam.na - Web: http://www.unam.na Oshakati Campus

APPENDIX L: CONSENT TO PARTICIPATE IN RESEARCH

UNIVERSITEIT•STELLENBOSCH•UNIVERSITY
jou kennisvennoot • your knowledge partner

[**The effect of a PBL approach on the teaching and learning of composition and inverses of functions in a Bridging course for undergraduate mathematics**.

SIGNATURE OF RESEARCH SUBJECT OR LEGAL REPRESENTATIVE

The information above was described to [*me/the subject/the participant]* by [*name of relevant person]* in [*Afrikaans/English/Xhosa/other]* and [*I am/the subject is/the participant is]* in command of this language or it was satisfactorily translated to [*me/him/her*]. [*I/the participant/the subject]* was given the opportunity to ask questions and these questions were answered to *[my/his/her]* satisfaction.

[I hereby consent voluntarily to participate in this study/I hereby consent that the subject/participant may participate in this study] I have been given a copy of this form.

____

Name of Subject/Participant

____

Name of Legal Representative (if applicable)

____ ________________

Signature of Subject/Participant or Legal Representative **Date**

SIGNATURE OF INVESTIGATOR

I declare that I explained the information given in this document to __________________ [*name of the subject/participant]* and/or [his/her] representative ____________________ [*name of the representative]. [H*e/she] was encouraged and given ample time to ask me any questions. This conversation was conducted in [*Afrikaans/*English/*Xhosa/*Oshiwambo*Other*] and [*no translator was used/this conversation was translated into* ___________ by _______________________].

____ 20 March 2013

Signature of Investigator Date

APPENDIX M: DESC APPROVAL LETTER STELLENBOSCH

UNIVERSITEIT•STELLENBOSCH•UNIVERSITY
jou kennisvennoot • your knowledge partner

Approval Notice
New Application

30-Sep-2013
CHIRIMBANA, Moses

Proposal #: DESC_Chirimbana2013
Title: The effect of a PBL approach on the teaching and learning of composition and inverses of functions in a Foundation Programme.

Dear Mr Moses CHIRIMBANA,

Your DESC approved **New Application** received on **19-Aug-2013**, was reviewed by members of the **Research Ethics Committee: Human Research (Humanities)** via Expedited review procedures on **27-Sep-2013** and was approved.

Please note the following information about your approved research proposal:

Proposal Approval Period: **27-Sep-2013 -26-Sep-2014**

Please take note of the general Investigator Responsibilities attached to this letter. You may commence with your research after complying fully with these guidelines.

Please remember to use your **proposal number** (DESC_Chirimbana2013) on any documents or correspondence with the REC concerning your research proposal.

Please note that the REC has the prerogative and authority to ask further questions, seek additional information, require further modifications, or monitor the conduct of your research and the consent process.

Also note that a progress report should be submitted to the Committee before the approval period has expired if a continuation is required. The Committee will then consider the continuation of the project for a further year (if necessary).

This committee abides by the ethical norms and principles for research, established by the Declaration of Helsinki and the Guidelines for Ethical Research: Principles Structures and Processes 2004 (Department of Health). Annually a number of projects may be selected randomly for an external audit.

National Health Research Ethics Committee (NHREC) registration number REC-050411-032.

We wish you the best as you conduct your research.

If you have any questions or need further help, please contact the REC office at 0218839027.

Included Documents:
Permission letters
Research proposal
DESC form

Sincerely,

Susara Oberholzer
REC Coordinator
Research Ethics Committee: Human Research (Humanities)